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Applying Game Theory in Invenory Control Problems

***Péter Mileff*[1]*, Károly Nehéz*[2]**
[1]PhD student, [2]PhD,
Department of Information Engineering, University of Miskolc

Abstract

The inventory control is a critical problem of the management of supplier companies for several decades. In recent years numerous new supply chain and inventory control models have been developed to support management decisions. In this paper, we investigate inventory holding and producing problems of supply chains with the method of game theory. We set up a two-person, non-cooperative finite game theory model for solving the classical one-customer and one-supplier problem. Our basic aim is to determine that strategy, which defends supplier from states of being a loser issuing from market uncertainty. We show, that the equilibrium of mixed strategy deviates from the analytic optimal solution.

Keywords: *Inventory Control, Supply Chain, Stochastic Demand, Game Theory*

1. INTRODUCTION

In successful inventory control models, critical inventory and cost-optimal inventory policy are realized by decisions. In the course of these, decisions are made about the starting time and the quantity of production. Naturally individual decisions involve many responsibilities which consequences are appeared in producing, logistical and business costs.

In the course of non-reusable and overstock product producing stock finance and inventory holding costs and in case of non-sufficient product producing penalty (back-order) costs are appeared. Modelling these latter is very difficult. Of course, the different models can possess different objective functions, and by means of this common interest can be realized between the end-product manufacturer and the supplier (common cost function). As constraint the strict non-admittance of the „lack" (e.g. short cycle JIT) can be appeared. In this paper we consider a model, which in general allows the risk of the back-order, but the frequency of these can be reduced to an optional small level by increasing with the penalty costs.

The literature decomposes the explanation of the penalty costs into three areas. According to the first explanation the supplier pays penalty cost in the course of back-orders, which means lost business. This case can be noticed by the simple „cool" buying-selling relation. The second explanation of the penalty cost shows that in the course of the back-order at the supplier there are not lost business, only penalty cost.

This approach supposes already some kind of „warm" relation between the business partners. The third area of the explanation possesses the feature of the first two explanations. In case of unsatisfied order, the big back-order volumes lead not only to penalty costs, but to losing business too. This relation of the business partners is likewise between the „cool" and „warm" relation. In this paper we refer the explanation of the penalty cost of the model to the second type.

We have chosen a game theory model for solving the inventory control problem of the supplier, which is based on cyclic demand of delivery and transport. The optimal stockpiling policy holds the supplier related costs at the minimal level on a long view. By minimizing the costs the profit maximization can be attained [6].

1.1 APPROACHING METHOD FOR THE SOLUTION OF THE PROBLEM

Nowadays gains greater and greater ground the game theory principled approaching methods of the solution of inventory control problems (Supply Chain Game). Game theory goes back to the long past, but only nowadays have been started to apply for modelling relations between the members of supply chain. The method is not only effective, but because of relation of problem it is very interesting too. Our aim in present paper is to apply the game theory to solve inventory problems, whereas determine of optimal inventory level can be explained an n-person, non-zero sum and simultaneous steps game. In this instance we restrict the problem only to the two-person, non-cooperative game category, in which the role of two partners consists of market and supplier.

We investigate the game from the supplier side and our objective is to find the optimal strategy, in which the supplier incurs a minimum loss. The inventory level of the game theory solution differs from the optimum. The cause is the defence against the large loss.

Both partner play with strategies in the game and try to choose one, which gives minimum loss against the supposed best choice of the other partner. Then it is said that the two strategy pair are the NASH equilibrium point of the game. A point from where no one of partners should not move, because his cost will be increased. However the NASH equilibrium of the games not absolutely means, that who have made the decision gains the most.

We assume that the supplier possesses only the lower and upper boundary values information of the expected demand. Moreover the supplier supposes that the market is malicious and chooses hostile strategies. Relying upon this findings we represent the minimalization of loss in the model as if market mixes two strategies randomly. He plays with D_{min} or with D_{max}. So the supplier must choose the inventory control policy, which does justice to this duality with the minimal loss. If he produces a lot, he satisfies the order, but his total cost will be increased by holding costs. On the other hand if he produces few, then plays with the risk of back-order.

2. Supply Chain Game

For the game theory solution of the supply chain we use the following supplier cost function [1]:

$$K_{sz}(q,D) = c_f + c_v(q-x) + p[max(D-q,0)] + h[max(q-D,0)]; \qquad (1)$$

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The game could be realized as a zero-sum game, if the loss of one of the partners would mean the profit of the other partner. Nevertheless in this instance with the meaning of the costs it is not fully true; therefore we introduce the cost function of the market (customer) as follows:

$$K_r(q,D) = c_s + c_r[\min(D,q)]. \quad (2)$$

In the cost function c_s means some kind of fixed cost (e.g. transport), still c_r is the cost of the product in peaces. The mean of $\min(D,q)$ is the quantity, which is possessed by the customer after transacting business. If $q > D$, then the quantity in compliance with the demand will appear at the customer. If $q < D$, then just as many as quantity are in the supplier inventory.

Let sign the set of players with $N = \{1,2,...n\}$, where in our case $|N| = 2$. Each player possess a not null set of the decision alternatives, let sign this $S_i = \{s_1, s_2, ..., s_n\}$. Examine the strategy set of the customer:

We have mentioned, that the market plays with two strategies. So his strategy set contains only these. Both of strategies mean a quantity of each discreet ordering product, which arrive to the supplier as a fixed producing period noted in the model. The strategy set of supplier also contains finite element discreet strategies, which mean the producing quantity series, which will be started by the supplier. The number of strategies equal the difference of the lower and upper demand bound.

Suppose that the game is simultaneous, where the players make their decisions at the same time in a such way that they choose a strategy from the strategy set independently from one another [3]. The players possess preferences in relation to the possible outcomes. These are represented with utility (payoff) functions explained on the $S = S_1 \times S_2 \times ... \times S_n$ product-set in the following manner:

$$u_i : S \rightarrow R, \text{ where } i = 1,\dots,n.$$

In many cases the payoff functions cannot be measured by numbers. But now the cost function itself will be the with utility (payoff) function [2]. If it would be a zero-sum game, then as we have above mentioned, it would be enough to investigate one, only the supplier cost function. Because the the customer and supplier cost function does not coincide with one another, therefore by reason of the two cost function we call a utility function regarding the supplier side into beeing. This utility function arises from the difference of the two cost function in the following manner.

$$H(s,D) = K_{sz} - K_r, \quad H : S \rightarrow R.$$

This function can be explained as follows: Entering the function we trace the game back to zero-sum game, where costs are interpreted with minus sign. In this explanation the cost, which is appeared at the customer side and realized as negative value, is realized in the utility function as the benefit of the supplier both in positive or negative meaning. So the bigger is the cost of the customer, the bigger is the benefit of the supplier. Every finite game can be specified with an n dimension array (polimatrix). Relying upon these findings the polimatrix of the utility function is the following:

Table 1. The strategy matrix of the supplier problem

S1\S2	***D_{min}***	***D_{max}***
S_1	$H(s_1,D_{min})$	$H(s_1,D_{max})$
S_2	$H(s_2,D_{min})$	$H(s_2,D_{max})$
..	..	..
s_n	$H(s_n,D_{min})$	$H(s_n,D_{max})$

Where $s_i \in S:\{q_1, q_2, ..., q_n\}$. Because the strategy set of both partner, and the number of the players are finite, so the game is called finite.

In the matrix $H(s_i,D_{min})$ means the game, when the market plays the D_{min} strategy and supplier plays the s_i strategy. Values (payoffs) in the matrix are the benefits of the supplier. Investigating these values the conclusion can be drawn; there are not dominant strategies in the game. So there are not a $s_k \in S$ strategy, in case of this the following ralation does not come true:

$$H(\hat{s}_i, D_{min}) \geq H(s_k, D_{min}) \text{ és } H(\hat{s}_i, D_{max}) \geq H(s_k, D_{max}),$$

where $\hat{s}_i \in S$ means the dominant strategy for the supplier.

The lack of dominant strategies results, that the game does not have unambiguous equilibrium point in the game played in the meaning of pure strategies. So applying mixed strategy game we consider, that the game would be played many times and it is allowed the payers to choose randomly from his strategies. Then their strategy sets are sum of strategy vectors explained on the original strategies, while their payoff functions will be the probabilities explained expected payoffs derived from the chosen strategy profile. Thus we call this obtained game the mixed expand of the original game. In the reality of course it is so, because if the demands of delivery are ensured always at a fixed time, then every time of demand of delivery can be explained as a new game. By playing the game repeated the average benefit can be maximized, which results minimalization of the average loss [2]. The following figure shows the utility function as function as it's two variables:

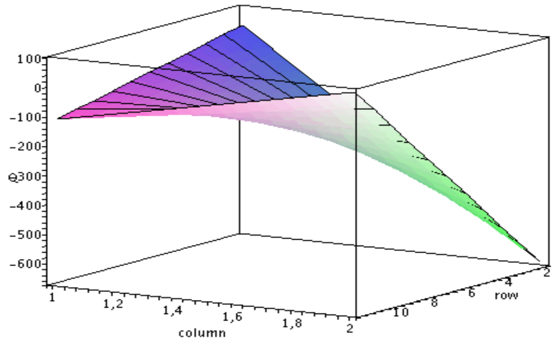

Figure 1. Supplier utility function

In this paper we do not discuss the analytical proof of the game theory method. The mixed strategy game theory solution offers generally more solutions. These appear as in the form of probability values, which mean strategies. All obtained probabilities imply that what frequency must be applied the given strategy. The game theory solution supposes none of producing supporting information. Therefore the obtained values always mean the product quantity that must be in the inventory at the time of ensuring the demand of delivery.

3. The Simulation

In the following we perform the 52 weeks simulation of the relation between the supplier and the market with help of MAPLE software package. We fix the parameters applied in the course of test as follows:

The demand of the product follows uniform distribution, in *10* number/week (D_{min}) and *20* number/week (D_{max}) interval. The demands are randomly generated as a uniform distribution between the boundaries. The back-order cost is $p = 70$ unit in case of every element, and the variable cost is $c_v = 10$ unit. The holding cost is $h = 5$ unit/period. The fix part of the production cost is $c_f = 30$ unit/series. The initial inventory level is II=0.

In the course of producing regarding 52 weeks we generated randomly the distribution of the produced quantity accordance with the obtained mixed strategies on the bases of probabilities. The following figure shows the results of the simulation:

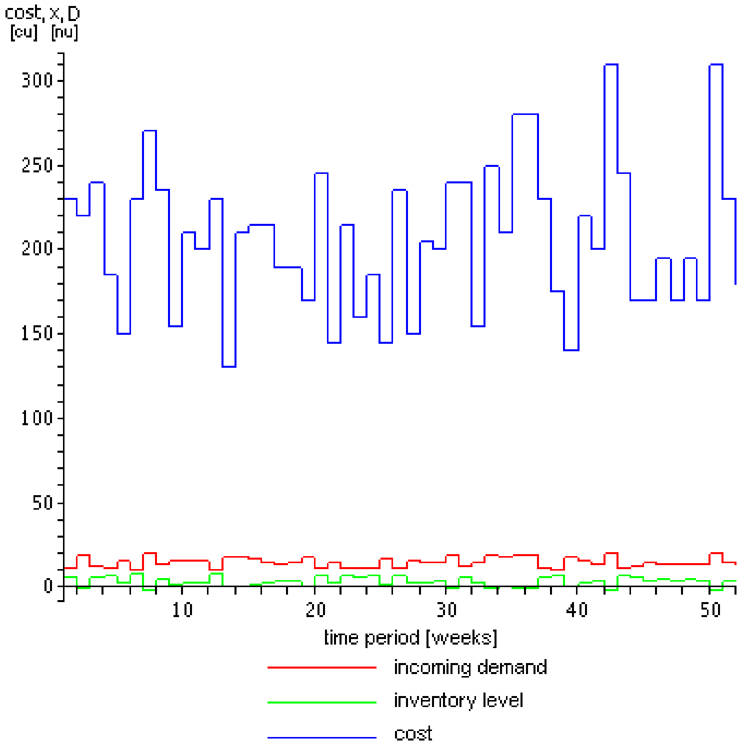

Figure 2. The 52 weeks simulation of the supplier problem

Lack and with this penalty cost occur that time, when the supplier inventory level is negative, which means back-order quantity too. As we can see, the demands are almost satisfied in 100%, only in some instances can be seen penalty cost. Increasing the value of the penalty parameter of course the probability of back-order can be decreased to zero [1].

In the next we compare the game theory model with the analytical solution of the publication [1]. The applied parameters and demands are naturally the same. Examine the following figure. This contains the cost of the two solving method in case of same demands.

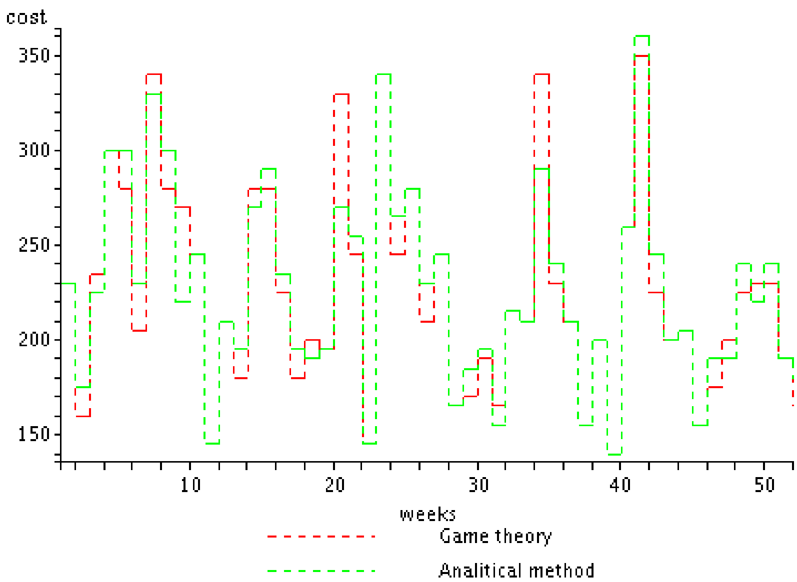

Figure 3. Cost comparison of inventory policies.

The figure shows properly, that in case of the same demands and parameters the game theory solution offers good solution similar to the analytical. In the course of simulations we observed, that the result of the 52 weeks simulation regarded to the two solutions show about 1% deviation. The cause of the difference is that on the basis of the obtained probabilities using mixed strategies, how to divide the quantity accordance with the 52 weeks strategies. There are such cases, when the game theory solution gives better result and there are cases when the analytical model has better result.

4. CONCUSIONS

In this paper we described the two-person game theory solution of the supplier inventory problem. We established a model, in which we investigated the relation of the two partners (supplier, customer) from the side of supplier. Accordance with cost function of the partners we set up the supplier utility function of the game, what was characterized with the lack of dominant strategies. These lacks led to the mixed strategies, which possibility of application was explained in later. The correctness of the model was justified and illustrated by a 52 weeks simulation as a function of fixed parameters. In the last part of the publication we compared in the course of 52 weeks simulation the game results with the results of the analytical supplier model described in the [1] publication. With help of this we justified the efficiency of the game theory.

5. FUTURE WORKS

For reason of further expansion of the model our aim is to develop a game theory solution for that case, when the supplier possess some kind of probability and forecast information regarding to some kind of time horizon. Justifying the correctness of the solution we compare the simulation results again with the forecast based analytical solution. Our objective is moreover to elaborate the competing games between the suppliers, where they can lose their customer.

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