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Applying Analitical Methods in Invenory Control Problems

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Abstract

The inventory control is a critical problem of the management of supplier companies for several decades. In recent years numerous new supply chain and inventory control models have been developed to support management decisions. In this paper, we investigate the classical one-customer and one-supplier problem with an analytical, event oriented model. Our basic aim is to determine an optimal inventory holding and production policy for suppliers, which means determining of an optimal and a critical inventory stock-level. The expected (average) cost of supplier using the optimal policy will be minimized under stochastic customer demands. We examine this problem by means of an own simulation method and analysis of the results will also be discussed.

Keywords: Inventory Control (IC), Supply Chain Management (SCM), Stochastic Demand

1 Introduction

In successful inventory control models, the critical inventory and the cost-optimal inventory policy are realized by decisions. In the course of these, decisions are made about the starting time and the quantity of the production. Naturally the individual decisions involve many responsibilities which consequences are appeared in producing, logistical and business costs. In the course of non-reusable and overstock product producing stock finance and inventory holding costs and in case of non-sufficient product producing penalty (back-order) costs are appeared. Modelling these latter is very difficult. Of course, the different models can possess different objective functions, and by means of this common interest can be realized between the end-product manufacturer and the supplier (common cost function). As constraint the strict non-admittance of the „lack” (e.g. short cycle JIT) can be appeared. In this paper we consider a model, which in general allows the risk of the back-order, but the frequency of these can be reduced to an optional small level by increasing with the penalty costs.

The literature decomposes the explanation of the penalty costs into three areas. According to the first explanation the supplier pays penalty cost in the course of back-orders, which means lost business. This case can be noticed by the simple „cool” buying-selling relation. The second explanation of the penalty cost shows that in the course of the back-order at the supplier there are not lost business, only penalty cost.

This approach supposes already some kind of „warm” relation between the business partners. The third area of the explanation possesses the feature of the first two explanations. In case of unsatisfied order, the big back-order volumes lead not only to penalty costs, but to losing business too. This relation of the business partners is likewise between the „cool” and „warm” relation. In this paper we refer the explanation of the penalty cost of the model to the second type. We have chosen an event-oriented model for solving the inventory control problem of the supplier, which is based on cyclic demand of delivery and transport. The optimal stockpiling policy holds the supplier related costs at the minimal level on a long view. By minimizing the costs the profit maximization can be attained [1].

We suppose the demand in the model is known as a random probability variable with its distribution function (in the simplest cases it is uniform distribution between a pre-defined D_{min} és D_{max} values). The model can be applied for other distributions too. The demands arrive as pre-known, fixed periodicity to the supplier. The knowledge of the distribution function is needed for the first step of the method. At this step in respect of a well-defined time interval we determine an optimal inventory level retailed on a long view in terms of the costs. In the later as the second step, we aim a less inventory level than this, using the information of the demand of delivery. To achieve this, a critical inventory level was determined, where the production and the non-production costs respecting to one time interval are equal. The third important step of the method is the decision, when a decision is born about starting a production run. If the current inventory level is less than the critical level, than production must be started, otherwise nothing has to do.

Our model examines the problem from the side of supplier. After the solution of the model we make known the steps performing the optimal policy.

1.1 THE COST FUNCTION

On the basis of the problem outlined in the model the supplier cost function regarding one time period can be formulated as a function of the parameters in the following manner:

$$K(q) = c_f + c_v(q - x) + pE[max(D - q, 0)] + hE[max(q - D, 0)]; \quad (1)$$

Where the individual parameters are the following:

- c_f – fix cost. This cost is always exist, when the producing of one series are started. [Ft / production]
- c_v – variable cost: This cost type means the production cost of one product. [Ft / product]
- p – penalty cost (or back order cost). If there is less raw material in the inventory then as much as satisfy the demands, this is the penalty cost of the unsatisfied orders. [Ft / product]
- h – inventory and stock holding cost. [Ft / product]
- D – It means that the demand from the receiver for the product, which is an optional probability variable. [number / period]
- E[x] – Expected value of the x stochastic variable.
- q – The product quantity in the inventory. The decision of the inventory control policy concerns the product quantity being in the inventory after the product decision. This parameter includes the initial inventory as well. If we don't produce anything, then this quantity equals with the initial, i.e. concerning the existing inventory.

x – Initial inventory. We assume that the supplier possesses x product in the inventory at the beginning of the demand of delivery period.

m – It means the effective producing quantity in the current time period. Its value is the difference between the optimal and the remained quantity in the last time period.

The first part of the equation expresses, that starting the production of every series carry some fixed costs, which expresses the starting cost of a new production run. The second part shows the variable costs of the products, which will be produced. Because we assume that *x* product is available in the inventory, therefore one technologic decision will result producing of *m=(q-x)* product. The third part of the cost function is the so-called penalty cost (back-order), which symbolizes the costs issuing from unsatisfied demands. The *max(x,0)* function performing in the cost function will be different from zero, if the demand is bigger than the quantity in the inventory. Of course it is possible events, when back-order is not allowable. This can be considered such as the *p* parameter of the model gets a high value. The last part of the equation gives the holding cost, which is arisen at that time when the demand was less than the quantity of the end-product in the inventory. If the demand is more than the produced quantity of the end-product, then of course there are no additional charges, because the inventory will be empty after filling the orders.

1.2 DETERMINING OF OPTIMAL STOCK LEVEL

On the basis of the above mentioned cost function the determination of the optimal inventory level is a minimization problem.

$$\frac{dK(q)}{dq} = \frac{d}{dq}\left(c_f + c_v(q-x) + pE[max(D-q,0)] + hE[max(q-D,0)]\right) = 0; \qquad (2)$$

From the upper relation for the $q = S$ optimal value (after a long derivation) we got an indirect solution. The complication rises from the handling of the operators of E expected value and max function. On a long view the amount of cost-optimal end-product can be calculated on the basis of the following relation:

$$F(S) = \frac{p - c_v}{p + h} \qquad (3)$$

where *F(D)* is the distribution function of the demand.

The value of *S* expresses the amount of the end-product should be in the inventory when the demand appears. It is easy to see, if there are not available the demand meeting amount, then necessarily should not be started a production, because this process carries such fixed cost, which makes the production of the small volume expensive. Consequently it is conceivable, that there certainly exist a critical amount, which is smaller then the optimal (*S*) amount, but choosing this quantity it is more profitable to sustain the risk of the back-order. The name of that point, where the cost of the decision about producing and the decision about non-producing and the decision where we rather undertake the risk of the back-order is equal, is the critical inventory level.

After these, the objective is to determine the critical level, which is probably smaller than the long-term cost-optimal inventory level. If the products in the inventory are less than this, only than is it profitable to increase the stock in hand to the optimal level. In the following we show how this level can be determined:

Let us introduce with nomination $L(q)$ the truncated, risk cost function, which is sum of the back-order and the inventory cost.

$$L(q) = pE[\max(D-q,0)] + hE[\max(q-D,0)]. \tag{4}$$

Suppose that the initial inventory level is less then the optimum level, namely $x < S$. If we increase this level to the optimum, then the total cost is:

$$K(S-x) = c_f + c_v(S-x) + L(S). \tag{5}$$

If the producing is not started, then on the other hand we have to calculate with only the initial inventory (x), so with $L(x)$. If the $L(x) \le K(S-x)$ condition is realized, then we don't have to produce, because the fix and the variable costs would increase the supplier cost. Determination of the critical inventory level can be attained with the $L(x) \le K(S-x)$ connection as follows:

$$L(x) \le c_f + c_v(S-x) + L(S), \tag{7}$$

$$L(x) \le c_f + c_v S - c_v x + L(S),$$

$$L(x) + c_v x \le c_f + c_v S + L(S).$$

Let s the critical inventory level. Our equation is modified as follows:

$$L(s) + c_v s = c_f + c_v S + L(S), \tag{8}$$

from where s can be determined. Understanding this solution, consequently if the inventory level of the supplier decreases under the critical level (s), then $m = (S - x)$ of quantity product must be produced. Otherwise it must not be produced, because the cost of the production is higher, than the cost of non-production.

4 SIMULATION METHOD FOR CHECKING THE POLICY

Henceforth we verified the solved work with help of 52 weeks simulation. We assume that the relation of the supplier and the customer is collaborative in the model. It means that in case of emerging of lack the supplier is liable for supplying the defect of the previous period in the next technologic cycle. Of this sanction and risk sharing are expressed by the contracting of clientele in the value of p. We applied MAPLE mathematical software package completion the simulation. The starting data of the illustrating example simulation are the following:

The demand of the product follows uniform distribution, in *10* number/week and *20* number/week interval. The back-order cost is $p = 60$ unit in case of every element, and the variable cost is $c_v = 10$ unit. The holding cost is $h = 5$ unit/period. The fix part of the production cost is $c_f = 30$ unit/series. The following figure shows the results of the simulation:

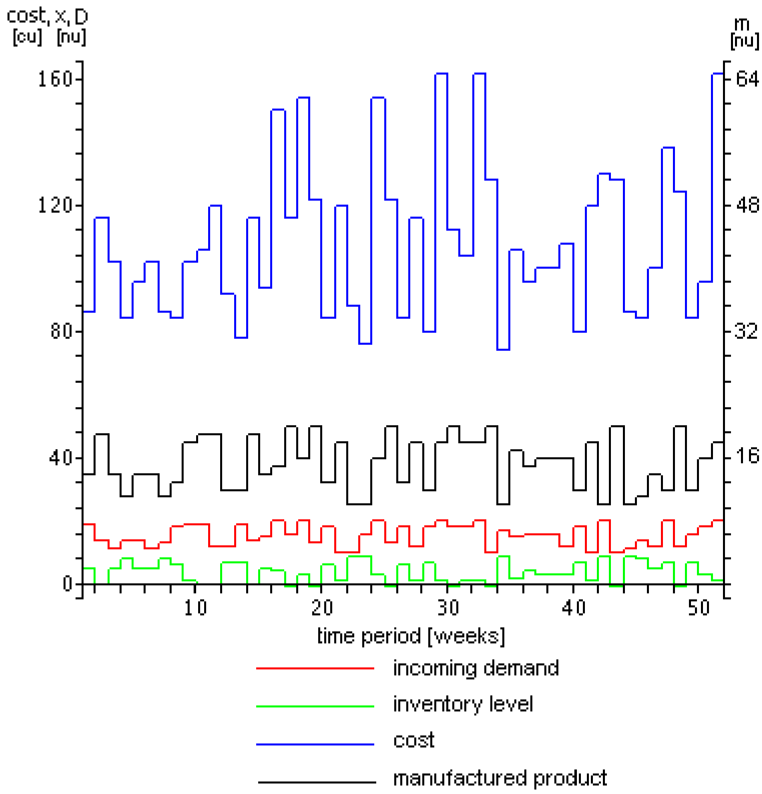

Figure 1. The simulation results of the supplier problem

Lack and with this penalty cost occur that time, when the supplier inventory level is negative, which means back-order quantity too. This happens when the stock is between the optimal and the critical inventory level, because at that moment the production is not started. If the incoming demand is bigger at this time, than the inventory level, therefore of course lack is arisen. At the real task the lack is not, or minimal allowable for one part of products. This can be built into the model, if we set a high value to p. The figure 2 shows how decreases the frequency of paying penalty cost to zero, while the penalty cost is increased.

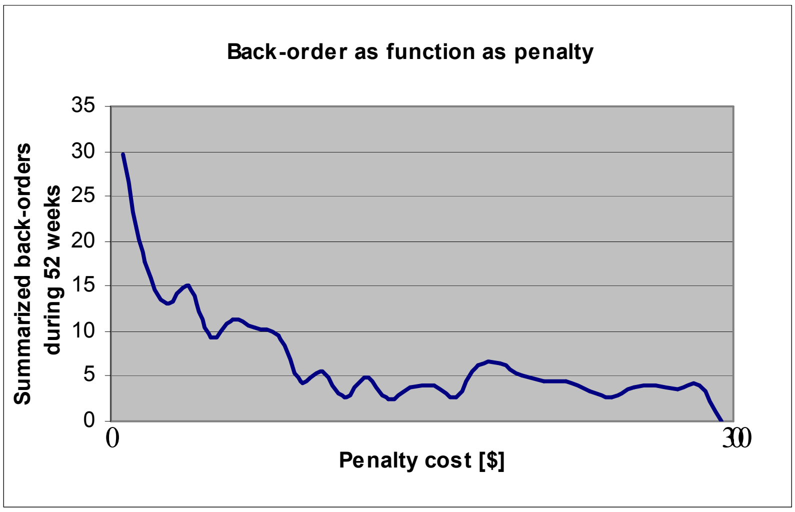

Figure 2. Back-orders and the penalty cost

5 CONCLUSIONS

In the present paper we examined the problem in case of the collaborative relation of one supplier and one customer. Improving the well-known models from the literature we optimized the cost function of the supplier as a function of the parameters, which does not eliminate the possibility of back-order. Understanding the problem as a non-linear optimization problem, we determined the optimal inventory level. To apply the correct inventory policy we introduced and defined the critical quantity of the inventory. To control the results we verified the correctness of the model with a 52 weeks simulation. With the help of this simulation we presented that the model with changing the parameters can be made suitable for attaining an optional low value of the lack respectively for transient observation of the changing inventory level. The model runs quickly on the simulator, therefore it is suitable for fast testing of different policies and decision alternatives.

Henceforth we aim at the expansion of decision work to the more periods. The objective of common investigation of several weeks is: determining all those week-pairs, where the aggregated production cost is less than the separately production cost. Thus with help of the received week-pairs, the total cost of the producing becomes still smaller. Further aim is to create a game theory model for the problem, which simulation results we would compare the obtained effects. It belongs to our aim to integrate the forecast information of the expected demand into the model, as well as to examine how historical data and uncertain forecast influences in time the conformation of inventory level.

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