Manufacturing-2006 Hungarian Scientific Conference. In memoriam: Joe Hatvany 80 years after his birth date, Budapest. pp. 141-149.

MODELLING AND SOLVING INVENTORY CONTROL PROBLEMS IN CUSTOMIZED MASS PRODUCTION

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Abstract:

Inventory control has been considered an essential problem in the management of supplier companies for several decades. In recent years numerous new supply chain and inventory control models have been developed to support management decisions. In this paper, we investigate the classical one-customer and one-supplier (newsvendor) problem with an analytical, event-oriented model. Our basic aim is to extend the classical newsvendor model to *n* periods, which means that management decisions will be made only once at the beginning of a predefined time horizon based on demand forecast information. The developed model is also suitable to handle the product run-out problem which arises from the seasonality of the real demand. A new heuristic method was developed to determine the optimal length of the time horizon. We examine the problem by means of a simulation method and discuss the results.

KEYWORDS: Supply Chain Management, Inventory Control, Newsvendor Model, Stochastic Demand

1. INTRODUCTION

The business environment of companies in the field of mass production has been altered in the last 15 years. The demand rate for mass products has remained at a high level but numerous new requirements have appeared on the market. The life cycle of products is becoming ever shorter. Customer needs for stylish forms, new modern designs, special packaging or better product properties have greatly increased. Generally, mass production companies assemble and bundle their products from components originating from their supplier companies.

Changes in the business environment influence engineering and logistic relations between companies and suppliers. The former, simple buying-selling (so-called “cool”) relation has become much “warmer”. This means that cooperative and collaborative methods and activities have become the main object in SCM development. The fast evolution of IT technology plays an important role in this process. In many respects, real-time, network-similar collaboration of independent, locally-separated companies is not realizable without an effective computer network information system. Relations of the marketing organizations, end-product manufacturers and supplier companies can be very complicated and various in practice. This motivates a wide examination of the available models and further investigation of effective decision supporting and planning methods.

Satisfying the fluctuating customer demands, multi-level stockpiling has been evolved. Warehouses of shopping centres, inventories of logistic centres, end- and semi-finished product warehouses of the end-product manufacturer, component and raw material warehouses of the suppliers are part of this chain.

In this paper we examine only the possibility of the component level stockpiling policy of suppliers. Inventory control has been considered an essential problem in the management of

supplier companies. To determine the inventory policy ensuring a compliant service level, numerous new models have been developed with modelling supply chains since years of 1950. In this paper, on the basis of claim of a Hungarian mass-production company, - stressing on the above outlined problem set - a new possibility of the cooperative stockpiling policy of the supplier is examined supposing stochastic demands.
We assume that estimations concerning the future (forecasts) have been solved; furthermore orders, acknowledgements, demands of delivery and the organization of the transport operation and synchronization of the planning process are also solved on the tactical level. We suppose that the supplier network on the strategy level is complete and bound by contract, and also that computerized communication conditions are available for the realization of business processes.
In this paper (at the current stage of the research), we consider the problem as a relation of one supplier and one end-manufacturer. Moreover we assume that the supplier produces one product and he is in relation to one end-manufacturer. The end-manufacturer can give a demand forecast at least for the middle-term, namely several weeks in advance (or other predefined time periods). But actual transport demand is given in the form of demand of delivery only a short time beforehand, typically one or two weeks. The information announced by the end-manufacturer is uncertain to some extent. The values of forecast and actual demand of delivery naturally do not coincide at all times.
Our basic aim is to determine a supplier stocking-producing policy, which makes cost-optimal stockpiling possible for an optional-length producing time horizon. To determine the appropriate inventory control policy, at first we use the one-period, stochastic newsvendor model, which is already well-known in the literature. The model is certainly among the most important models in the field of operations management. It is applied in a wide variety of areas: centralized and decentralized supply chain inventory management (e.g., Shang and Song 2003, Cachon 2003), retail assortment planning (e.g., van Ryzin and Mahajan, 1999), international operations (e.g., Kouvelis and Gutierrez 1997), horizontal competition among firms facing stochastic demand (e.g., Lippman and McCardle, 1995), lead time competition (e.g., Li 1992), outsourcing and subcontracting decisions (e.g., Van Mieghem 1999), product and process redesign (Fisher and Raman 1996 and Lee 1996), and spot markets and inventory control (e.g., Lee and Whang 2002).
Our model examines the problem from the point of view of the supplier. The collaborative interests appear in the model through the parameter values. After the solution of the model we review the steps performing the optimal policy.

2. ONE-WEEK COST FUNCTION (CLASSICAL NEWSVENDOR MODEL)

The classic newsvendor model considers a type of problem that many decision makers (newsvendors) encounter in the business world. Facing uncertain demands for limited-useful-life products (such as mobile phones, fashionable goods etc.), a decision maker (newsvendor) needs to decide how many units of these goods to order for a single selling period. Intuitively, if she/he orders too many (surplus), this may cause unnecessary inventory cost. Thus, the cost will be too high. Whereas, if the decision maker (newsvendor) orders too few (shortage), it will miss opportunities for additional profits because some customers have no chance to buy the goods. The optimal solution to this problem is characterized by a balance between the expected costs of shortage and surplus.
On the basis of the problem outlined in the model the supplier cost function regarding one time period can be formulated as a function of the parameters in the following manner [1][8]:

$$K(q) = c_f + c_v(q - x) + pE[max(D - q, 0)] + hE[max(q - D, 0)]. \quad (1)$$

where the individual parameters are the following:

c_f – fixed cost. This cost always exists when the production of a series is started. [Ft / production]

c_v – variable cost. This cost type expresses the production cost of one product. [Ft / product] *p* – penalty cost (or back order cost). If there is less raw material in the inventory than needed to satisfy the demands, this is the penalty cost of the unsatisfied orders. [Ft / product]

h – inventory and stock holding cost. [Ft / product]

D – This means the demand from the receiver for the product, which is an optional probability variable. [number / period]

E[D] – Expected value of the *D* stochastic variable.

F(D) – cumulative distribution function of *D*

f(D) – probability mass function of the demands

q – The product quantity in the inventory. The decision of the inventory control policy concerns the product quantity in the inventory after the product decision. This parameter includes the initial inventory as well. If nothing is produced, then this quantity is equal to the initial quantity, i.e. concerning the existing inventory.

x – Initial inventory. We assume that the supplier possesses *x* products in the inventory at the beginning of the demand of the delivery period.

m – This means the effective producing quantity in the current time period. Its value is the difference between the optimal and the remaining quantity from the last time period.

d – Expresses the loss per unit of left over goods. Using in case of production run-out.

λ – Parameter of the Poisson distribution.

On the basis of the above-mentioned cost function the determination of the optimal inventory level is a minimization problem [9][1]. So in the long run the amount of cost-optimal end-product can be calculated on the basis of the following relation:

$$F(S)=\frac{p-c_v}{p+h}=\frac{p-c_v}{(p-c_v)+(h+c_v)}\text{, thus} \qquad (2)$$

$$F(S)=\frac{shortage\ \cos t}{(shortage\ \cos t)+(surplus\ \cos t)}\longrightarrow P(D\le q)=\frac{P(D\le q)}{P(D\le q)+P(D>q)}, \qquad (3)$$

where *F()* is the distribution function of the demand. The value of S expresses the amount of the end-product that should be in the inventory when the demand appears. In the optimality condition $\xi:=(p-c_v)/(p+h)$ is known as the critical fraction. Also note that the critical fraction is the probability of not stocking out (somewhere known as Cycle Service Level [3]).

3. EXTENDING THE NEWS VENDOR PROBLEM TO *N* JOINT WEEKS

The optimal inventory policy of the supplier was discussed above in short projected for one production time horizon. Here, this policy will be developed in such a way that it is to be suitable to cover uniformly *n* number production cycles (weeks). Then we demonstrate the determination of the optimal inventory level in the course of the joint production of *n* number of weeks. With the help of these optimum values it becomes possible to determine the necessary production cycles in a specific time horizon. This optimal cycle number, which means the minimal setup number, opens the door to minimize the costs on the supplier's side.

3.1 Determining the inventory optimum for *n* joint weeks

Let us extend the "newsvendor" problem to *n* joint weeks. This means that we do not produce every week but we try to determine an optimal number of *n* joint weeks based on demand forecast information. Especially in mass production, where setup costs and production rate are high values and holding cost is low, it is not worth producing every week. We suppose that the necessary production capacities are available.

Nevertheless holding and penalty costs may arise each week, thus with these factors the new cost function becomes more complicated. The cost function of the joint production for *n* weeks is the following:

$$\begin{aligned}K_{123\ldots n}(q_{123\ldots n}) &= c_f + c_v\left[q_{123\ldots n} - x\right] + hE\left[q_{123\ldots n} - D_1\right]^+ + hE\left[q_{123\ldots n} - D_1 - D_2\right]^+ + \ldots + \\ &+ hE\left[q_{123\ldots n} - D_1 - D_2 - \ldots - D_n\right]^+ + pE\left[D_1 - q_{123\ldots n}\right]^+ + pE\left[(D_1 + D_2) - q_{123\ldots n}\right]^+ + \ldots + \\ &+ pE\left[(D_1 + D_2 + \ldots + D_{n-1}) - q_{123\ldots n}\right]^+ + pE\left[D_n + \left[\ldots + \left[D_2 + \left[D_1 - q_{123\ldots n}\right]^-\right]^-\right]^-\right]^+,\end{aligned} \quad (4)$$

where $q_{123\ldots n}$ means the quantity which appears in the supplier inventory early in the cycle for *n* week after production. $D_1, D_2, \ldots, D_n$ mean the demand of the specific week, which appears as a continuous random variable. It can be seen that the last part of the function is fairly complicated, but by means of several mathematical transformations [8] $min(D,q) = D - max(D-q,0)$, $min(D,0) = D^- = D - D^+$, $max\{D,q\} = D^+$, we obtain a simpler variation of (4):

$$\begin{aligned}K_{123\ldots n}(q_{123\ldots n}) &= c_f + c_v\left[q_{123\ldots n} - x\right] + hE\left[q_{123\ldots n} - D_1\right]^+ + hE\left[q_{123\ldots n} - D_1 - D_2\right]^+ + \ldots + \\ &+ hE\left[q_{123\ldots n} - D_1 - D_2 - \ldots - D_n\right]^+ + pE\left[(D_1 + D_2 + \ldots + D_n) - q_{123\ldots n}\right]^+.\end{aligned} \quad (5)$$

The difference between (4) and (5) is that the penalty cost was simplified because we exploited that fact the penalty is considered as an independent value during *n* periods. Therefore penalty cost can be calculated with the difference of the sum of demands and production quantity.

From the form of (5) it can be expressly seen that the problem is traced back to the one-week production problem. The solution, the optimal inventory quantity of the joint production for *n* weeks, arises as a minimization problem in the following manner:

$$F_{123\ldots n}(q_{123\ldots n}*) = \frac{p - c_v - hF_1(q_{123\ldots n}) - hF_{12}(q_{123\ldots n}) - hF_{123}(q_{123\ldots n}) - \ldots - hF_{123\ldots n-1}(q_{123\ldots n})}{(h+p)}, \quad (6)$$

where $F()$ represents the joint distribution function in compliance with the number of weeks drawn together. The $q_{123\ldots n}^*$ - which satisfies the equation - expresses that the finished goods must be in the inventory at the time when customer demand appears with regard to *n* weeks. (6) can be solved by using numerical methods but in special cases some simplification can be applied.

In practical calculations, values of $hF_1(q_{123\ldots n}), hF_{12}(q_{123\ldots n}), hF_{123}(q_{123\ldots n}), \ldots, hF_{123\ldots n-2}(q_{123\ldots n})$ can be approximated by 1. Note that: assuming that *F1(x) is a* cumulative distribution function of a uniform distribution. If argument *x* is greater than the maximum value of the given uniform distribution, then *F(x)* always gives 1 by definition. This way equation (6) becomes a simplified form as follows:

$$F_{123\ldots n}(q_{123\ldots n}*) = \frac{p - c_v - (n-2)\cdot h - hF_{123\ldots n-1}(q_{123\ldots n})}{h+p}. \quad (7)$$

The numerator can be a negative value in that case, when holding cost during the weeks is greater than a certain limit and in this case, naturally there is no optimal solution. This time the number of weeks have to be reduced, because it is cheaper if the supplier does not produce anything. Characteristic of CSL in case of joint production can be seen at figure 1.

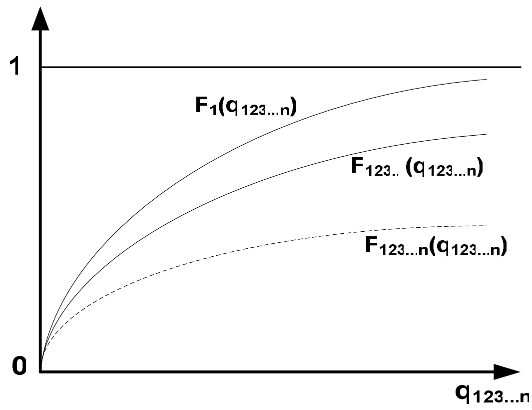

Figure 1: CSL characteristic of joint productions

Of course the critical inventory level [8] mentioned first by Herbert Scarf [11] for one-week production can be applied in the case of joint production for *n* numbers of production cycles. However this is not discussed in this paper.

3.2 Optimal inventory control policy for *n* weeks

As we mentioned above, the optimal inventory quantity in case of joint production for *n* weeks can be calculated with the relation (6) showing similar symmetry to the one-week production problem. In the following, the practical importance and of joint production for *n* weeks will be introduce in brief through a concrete example. The empirical factors show that the (high) value of the production fixed costs influences significantly the number of setups in the specific production time horizon, and indirectly the costs. The greater this value is, the less profitable production cycle usage is in the specific order time window. Thus cost referring to the specific time horizon will be minimal if and only if the number of setups is minimal as a function of production fixed costs. In the following, both the optimal number of setups and the optimal production quantity can be determined. The method uses directly the above-mentioned cost function of joint production for *n* weeks.

We assume that the distribution functions of the probability random variables in the cost function are normal. The manner can be applied in case of optional distribution, but in this example we choose normal distribution because of easier lucidity. The reason is that the distribution and cumulative functions of the new variable (e.g.: $D_{123} = D_1+D_2+D_3$) established from the sum of uniform distribution probability random variable are complicated. Supposing normal distribution, the σ and μ values of this new variable is realized as the sum of the σ and μ values of the added variables. Moreover we assume that values of σ and μ are equal for all weeks.

The basic idea of the method is as follows: with the help of the cost function (4), the optimal production quantity can be determined regarding an optional time horizon. However this quantity is independent from production fixed costs, because the *n* number of weeks are considered as a production cycle. It is easy to see that relating to a certain time horizon the necessary number of setups depends greatly on the value of production fixed costs. Controlling this problem, we introduce the concept of per-unit cost, which means the cost per unit regarding a specific time horizon.

Let denote the value of per-unit cost $\hat{K}_i = K_i / q_i^*$, where K_i means the cost of *i*-th number of jointly produced weeks, q_i^* is the optimal quantity of *i* number of jointly produced weeks and $i = 1...k$. We suppose that the value of per-unit cost in case of different numbers of jointly produced weeks will be different. The objective is to find the minimal from this set. Thus finding the $\hat{K}_{min}$, which satisfies the following equation: $\hat{K}_{min} = min\{\hat{K}_1, \hat{K}_2, ..., \hat{K}_n\}$. Possessing the minimum shows clearly the necessary number of weeks in a specific time horizon needed for joint production. The necessary minimal number of setups arises already from this.

The first step of the method is using the formula of joint production for *n* weeks. In the course of this the optimum of *k* number of jointly produced weeks comes to determination. Experience shows that the value of *k* can be maximum 7 – 8 in practice. The determination of the optimum can be calculated analytically with the help of the above-mentioned method, which needs very little calculation time. We prove our assumption through the next practical illustrative example.

Distribution of demands for products are normal with 15/week mean value and with variation $\sigma = 3$. The back-order cost is p = 40 unit for each element. Holding cost is h = 2 unit/period. The fixed cost of the production is c_f = 120 unit/series. Let the value of variable cost c_v = 5 unit. The per-unit costs of the $k = 1...9$ number of jointly produced weeks and the produced optimal q quantities obtained in the course of computations are summarized in Table 1.

	Week 1	***Week 2***	***Week 3***	***Week 4***	***Week 5***	***Week 6***	***Week 7***
Optimal quantity	16.91364	*33.26343*	***49.16258***	*64.67296*	*79.83145*	*94.66017*	*109.17202*
Per-unit Cost	*12.68306*	*9.93307*	***9.78449***	*10.21825*	*10.86718*	*11.61334*	*12.40708*

Table 1: Results determining the optimal week number for joint production

The columns of the table mean the number of jointly produced weeks. The first row denotes the optimal quantity as a function of jointly produced weeks and the last row means the per-unit cost. For example, 79.83145 in the first row means that if we produced on the basis of 5-weeks forecast information, the optimal quantity would be 79.83145. In this case, 10.86718 is the per-unit cost of the production. Clearly, the per-unit cost is minimal in the third column (9.78449). With the help of Figure 2, per-unit cost can be studied as function of jointly produced weeks.

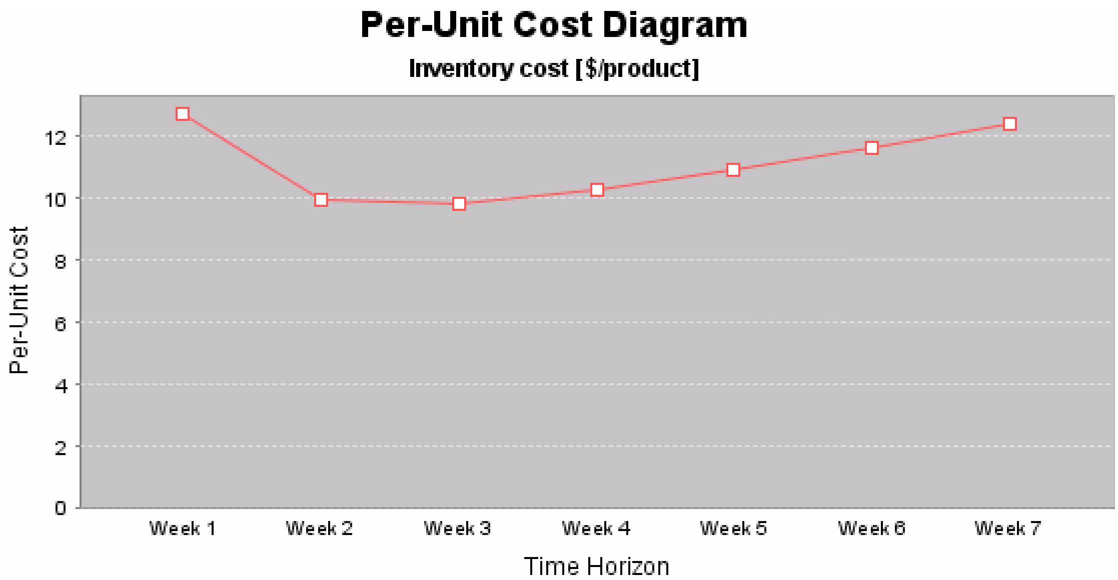

Figure 2: Variation of per-unit cost

The *x* axis in Figure 2. shows the production time horizon as function of weeks and the *y* axis represents the per-unit cost as function of jointly produced weeks. Once again, it is easy to see that the value of per-unit cost is minimal supposing joint production of a three-week (cycle). Using this minimal cycle number the costs will be really minimal.

Using this method the minimal number of required production cycles can be determined in a specific production time horizon, as can be the optimal quantity that needs to be manufactured in the cycle. We verified the accuracy of our heuristics using the method of constraint programming and with a genetic algorithm. Results prove clearly the efficiency of the new method. The enormous advantage of this manner is that it is substantially faster than brute force or a genetic algorithm. Since it is an analytical solution, therefore this will be always an exact solution or at least the error can be estimated in advance.

3.3. Interpretation of product run-out

Following the explanation of joint production of production cycles we change over to the run-out problem of the products, which has nowadays a great importance. Market estimations prove clearly that natural fluctuation can be observed at product demands. It means that a kind of so-called seasonality can be explained for products. According to this seasonality, demands for products change greatly in a certain period. Almost all larger or smaller commercial firms, retailers, suppliers come face to face with this problem. They try to defend themselves against this with help of human intuition and experience of previous years.

Of course human intelligence is essential in case of a complex problem. Our aim is to extend the model with new elements, which can help to solve the problem beside human intuition. Applying this extension the more exact computation of necessary inventory level becomes possible.

There cannot be found so many examples modeling the production run-out in the literature of the newsvendor models. In case of stochastic models, logistic distribution [Weisstein, 2006] is applied many times modeling these problems. In many publications the authors calculate the optimal inventory level decreasing in function of this method. The disadvantage of these methods is that the models loose its distribution function independency.

Our aim is to develop a model extension. With help of this, the problem of product run-out can be examined in case of pre-mentioned joint production of production cycles and the model keeps its distribution function independency. The terminology “run-out” expresses that the demand for a specific product will be reduced to zero after a certain time. There was given any notice of the demand from the forecast system, but the product suddenly ran out (E.g. packaging material produced for Christmas). It is looked for an element (function), which expresses this in time ascending risk factor. In compliance with this, the run-out of demands is modelled with a random variable with Poisson distribution function. In the following figure the Poisson distribution function can be seen in case of $\lambda = 15$.

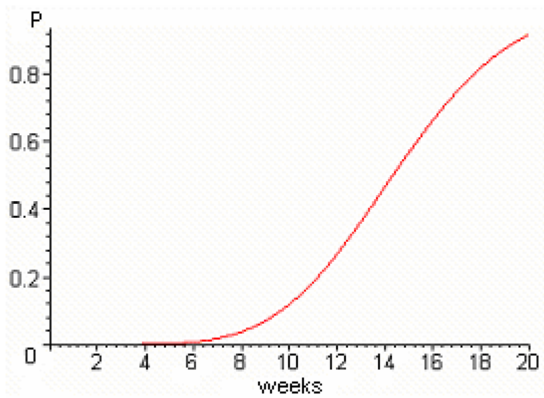

Figure 3: Poisson distribution function with $\lambda = 15$.

$\lambda = 15$ means that the event occurs once during 15 time period (week). Figure 3 shows that the probability of occurrence during the first 8 weeks is small ($P(\zeta < 8) \approx 0.066$). So the product runs out during the first 8 weeks with 6.6% probability. Applying two weeks joint production, the cost function with its introduced new parts can be represented as follows:

$$\begin{aligned} K_{12}(q_{12}) &= c_f + c_v\left[q_{12} - x\right] + hE\left[q_{12} - D_1\right]^+ + hE\left[q_{12} - D_{12}\right]^+ + pE\left[D_{12} - q_{12}\right]^+ + \\ &+ \boxed{dR(1,\lambda)E(q_{12} - D_1)^+ + dR(2,\lambda)E(q_{12} - D_{12})^+} \end{aligned}, \quad (8)$$

Where d is a positive real number and expresses the loss per unit of left over goods. $R(i,\lambda)$ is the Poisson distribution function, where i is a positive integer designating the weeks. $\lambda = [0..\infty]$ is the parameter of the Poisson distribution. On the basis of the above mentioned expressions:

$$\frac{dK(q_{12})}{dq_{12}} = c_v + hF_1(q_{12}) + hF_{12}(q_{12}) - p(1 - F_{12}(q_{12})) + dR(1,\lambda)F_1(q_{12}) + dR(2,\lambda)F_{12}(q_{12}).$$

The derivation is similar to the other solution in the appendix, so this is not discussed here. Thus the solution:

$$F_{12}(q_{12}*) = \frac{p - c_v - hF_1(q_{12}) - dR(1,\lambda)F_1(q_{12})}{h + p + dR(2,\lambda)}. \quad (9)$$

The problem can be explained similarly in case of N week joint production as well. Product run-out is interpreted for n number of weeks. The solution in this case: (It is not detailed here, but can be proved easily)

$$F_{123\ldots n}(q_{123\ldots n}*) = \frac{p - c_v - hF_1(q_{123\ldots n}) - hF_{12}(q_{123\ldots n}) - \ldots - hF_{123\ldots n-1}(q_{123\ldots n}) - \sum_{i=1}^{n-1}\left[dR(i,\lambda)\right]}{h + p + dR(n,\lambda)}. \quad (10)$$

In favour of practical feasibility, choosing values of d and λ have crucial importance. Value of d can be comprehended as penalty cost, which changes per units. λ expresses the seasonality of the product. So the smaller is its value, the larger is the danger of its run-out.

4. CONCLUSIONS

In the present paper we examined the problem in case of the collaborative relation of one supplier and one customer on the basis of claim of a Hungarian mass-production company. Improving the well-known models from the literature, we optimized the cost function of the supplier as a function of the parameters, which does not eliminate the possibility of back-order and is applicable for an optional time horizon. We extended the problem of a one-week production cycle to become the production of optional, n number of jointly produced weeks possible. Understanding the problem as a non-linear optimization problem, we determined the optimal inventory level. With the help of a heuristic method over and above the optimal inventory level, the minimal required number of jointly produced weeks can be defined in an exact way. The efficiency of the method was proved by a genetic algorithm and constraint programming. The simulation results show clearly that calculation time of the method is small; therefore it is suitable for fast testing of different policies and decision alternatives.

The developed model gives also a suitable solution to handle the product run-out problems as well, which arises from the seasonality of the real demand. Henceforth we aim to extend the developed model to be applicable multi-product problems with natural capacity constraints. Part of our aim is to integrate the forecast information of the expected demand into the model, as well as to examine how historical data and uncertain forecasts influence the conformation of inventory level over time.

5. ACKNOWLEDGEMENTS

The research and development summarized in this paper has been carried out by the Production Information Engineering and Research Team (PIERT) established at the Department of Information Engineering. The research is supported by the Hungarian Academy of Sciences and the Hungarian Government with the NKFP VITAL Grant. The financial support of the research by the aforementioned sources is gratefully acknowledged. Special thanks to Ferenc Erdélyi for his valuable comments and review works.

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