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SOLVING CAPACITY CONSTRAINT PROBLEMS IN A MULTI-ITEM, MULTI-PERIOD NEWSVENDOR MODEL

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Abstract

In the last few years, the effective management of the inventory control problems became ever more critical problem of the supplier companies. In this paper, on the basis of demands of a major Hungarian mass production company, an extension of an analytical inventory control model is presented considering the condition of global capacity constraint. This model was elaborated in our previous paper [7] and models the one customer - one supplier relation. Our aim is to determine an optimal holding-production policy of the supplier, which makes possible a cost-optimal stockpiling policy for an optional long production time horizon. We show, that building on our former results, the global capacity constraint satisfying policy can be determined by the help of a new heuristic method.

Keywords: Stockpiling policy, Extended newsvendor model, Global capacity constraint

1. INTRODUCTION

In the last 15 years, the business environment of companies in the field of mass production has altered. The demand rate for mass products has remained at a high level but numerous new requirements have appeared on the market. Changes in the business environment influence engineering and logistic relations between companies and suppliers. The former, simple buying-selling (so-called "cool") relation has become much "warmer". This means that cooperative and collaborative methods and activities have become the main object in SCM development. Relations of the marketing organizations, end-product manufacturers and supplier companies can be very complicated and various in practice. This motivates a wide examination of the available models and further investigation of effective decision supporting and planning methods [10].

In the literature we can meet the wide scale of stockpiling models [6]. In the later we deal with one of the most known stochastic method, so-called "newsvendor model". The model is certainly among the most important models in the field of operations management. It is applied in a wide variety of stockpiling problems. In this paper in compliance with this, an extended newsvendor model [7] will be examined on the basis of former results in a multi-product and capacity constraint case.

The model conditions are fully identical with the conditions presented in [5][6] publications. The greater part of the models in the literature solves the problem applying the tool of the dynamic programming or some kind of searching method (soft-computing). Because of the large searching space, these solutions require extremely long computation time in case of many product number and long production time horizon.

2. THE ONE PRODUCT, MULTI-PERIOD MODEL

Introducing the global capacity constraint of the multi-period extended newsvendor model, we consider the characteristic of the model as a 'week based' policy. Accordingly two types of optimization method can be differed:

- *Service Level based policy*
- *Cost based policy*

2.2 Cost based policy

This approach models that type of supplier, which is directly in relation to the market. In case of this type of policy, the main objective is to minimize the costs. It should be decided that how many back-orders can be allowed during the time horizon. So the penalty cost is determined by the supplier itself.

Because of production cost optimization, decreasing setup number or taking the penalty can be an alternative. The reducing of the number of jointly produced weeks is not definitely the best solution. It is possible that taking the penalty cost is cheaper for the supplier. Denote the cost value of the occurred back-orders with variable *b*. The next equation helps to decide what policy should be applied.

$$b + h \cdot a \cdot (C - Q_a) \le c_f \text{, otherwise } b + \sum_{i=1}^{a} h \cdot (C - Q_a) \le c_f \text{,}$$

where h is the holding cost and Q_a is the quantity in compliance with the number of jointly produced weeks. Variable a means the week number, which Q_a value is even smaller than the C capacity constraint. $h \cdot a \cdot (C - Q_m)$ represents the holding cost, which appears as difference of the quantity of the capacity constraint and the quantity of the reduced jointly produced weeks. The meaning of the formula: in the case, when the value of $b + h \cdot a \cdot (C - Q_m)$ is cheaper than a new setup cost (c_f), the cost will be minimal, when the supplier chooses to take the penalty and produces the quantity of the capacity constraint. Otherwise reducing the number of the jointly produced weeks is the good policy. Figure 1 shows the method of the cost based policy properly.

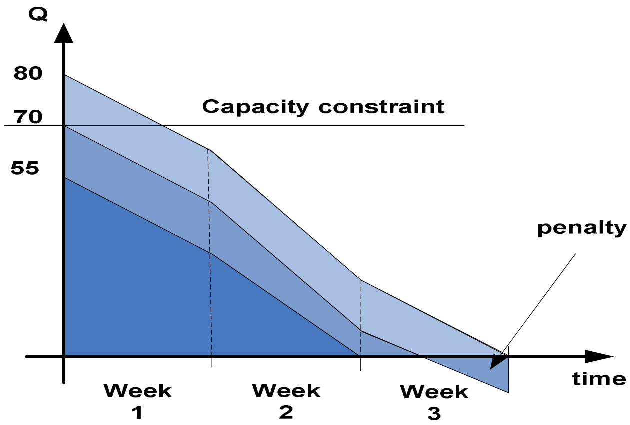

Figure 1. Applying capacity constraint in case of cost base policy

3. THE MULTI-PRODUCT, MULTI-PERIOD MODEL

Solving capacity constraint problems in case of this model can be complicated. We assume a global capacity constraint, which means that products share in one common capacity. In this presented solution we prefer the Service Level based policy. The objective is to determine the reduced number of jointly produced weeks per product in such a way that the sum of the total amounts satisfies the capacity constraint condition.

We start with the per-unit cost of the products. Figure 2 shows the variation of the per-unit cost in function of jointly produced weeks.

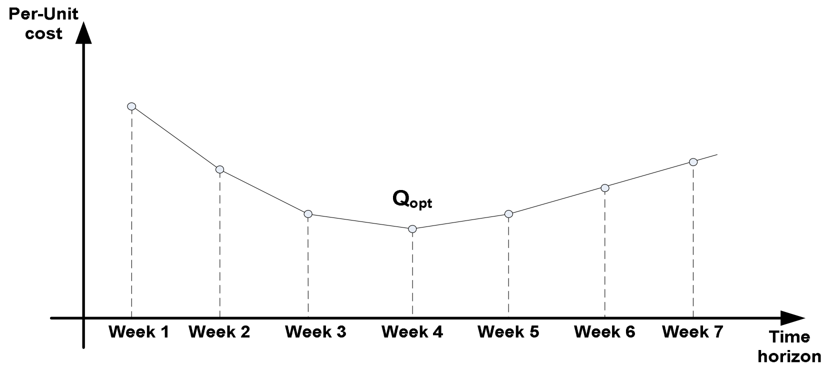

Figure 2. Per-unit cost variation in case of seven weeks length time horizon

Each product has a similar per-unit curve. If the sum of the quantities of *n* number of products is greater than the value of capacity constraint, then the solution should be modified. If

$$\sum_{j=1}^{n} u^j \cdot (q_i^j * - x^j) \le C, \text{ the solution is optimal.}$$

In the equation *j* (*j=1,…,n*) means the number of products, q_i^j is the optimal quantity of the product *j* in case of *i* (*i=1,…,m*) number of jointly produced weeks. x^j denotes the initial inventory of product j. The question is, which setup number of product should be modified?

Let us start from examination of the per-unit cost curve. The next figure shows the variation of the per-unit cost for 4 weeks based on the modification of Figure 3.

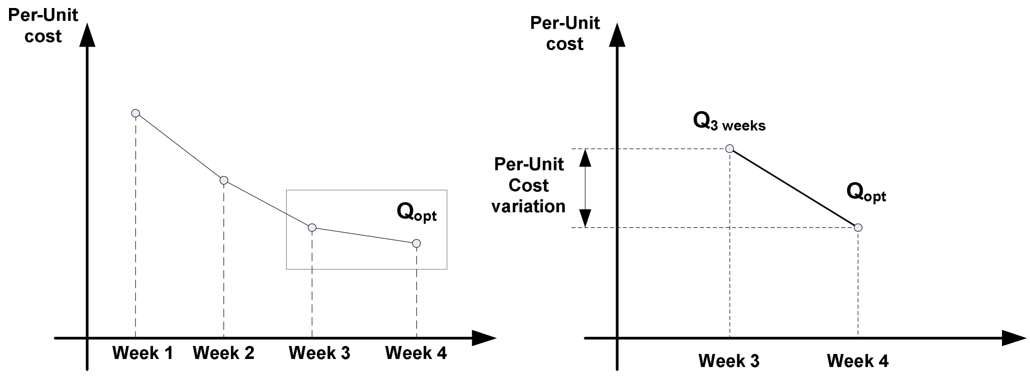

Figure 4. Increase of per-unit cost in function of jointly produced weeks

From figure 4 it is easy to see, if the optimal number(4) of jointly produced weeks is decreased to three weeks, the value of the per-unit cost will be certainly increased. The capacity constraint suitable solution should be determined in the way that the sum of reductions of the number of jointly produced weeks preformed at the products, will be minimal. The result completes the capacity constraint condition and has minimal cost. Denote $FSKV_j$ the sum of performed per-unit cost variations at product *j*. Then:

$$\sum_{j=1}^{n} FSKV_j \longrightarrow min\,.$$

3.1 The algorithm

In the following subsection, the solver algorithm will be shown. It consists of three main parts: 1. *Testing capacity constraint condition*, 2. *Choosing modifications, which have chance to be an optimum point*, 3. *Choosing merge combinations in order to have a better solution*.

Notations:

n	number of products.
opt_i	optimal number of jointly produced weeks of product *i*.
$q_{i,j}$	means the optimal quantity of product i, if j number of weeks is produced jointly
$FSK_{i,j}$	per-setup cost of product i produces j number of weeks jointly. Value: $FSK_{i,j} = \frac{c_{f,i}}{q_{i,j}}$.
$P_i[1...opt_i]$	help vector of product i. The value of element k: $P_i[k] = FSK_{i,k+1} - FSK_{i,k}$.
L[1...n]	solution vector. Initialized by zero [0…0]. Its element sign, how much weeks should be reduced at the products.
B[1...n]	boolean auxiliary vector. At the substitution phase, it helps to sign the occurred modifications at the products.
minFSKV	auxiliary variable. It stores the index of the product which has the minimal per-setup cost variation.
maxFSVK	auxiliary variable. It stores the index of the product which has the maximal per-setup cost variation.

C	value of the global capacity constraint.
Sum	auxiliary variable. In the substitution phase it stores the value of $P_i[opt_i - L[i]]$ in case of $B[i]=true$. ($i=1 \ldots n$).
$minIndex$	auxiliary variable. The index of the chosen element for substitution.

Start and Capacity constraint condition test

Step 1.: With help of per-unit cost model, we calculate the optimal number of jointly produced weeks of products (opt_i). The vectors $P_i[1...opt_i]$ are filled up with these values. Later these vectors store the values of per-setup cost variations.

Step 2.: The solution is tested to fit with the capacity constraint.

$$\sum_{j=1}^{n} q^{j}_{opt_i - L[j]} \le C \ .$$

If the condition is satisfied, then the solution is optimal. Otherwise jump to Step 3.

Choosing the optimum possible modifications

We choose the cases at this step, which can be solutions.

Step 3: Collect the indexes of the products, which have the minimal and the maximal value of per-setup cost variation. Thus: Minimal: $minFSKV = min\, P_i[opt_i - L[i]]$. Maximal: $maxFSKV = max\, P_i[opt_i - L[i]]$. Note that: the maximum index is calculated only at start and after substitution. Then $L[minFSKV]=L[minFSKV]+1$ and $B[minFSKV] = true$;
The product is a possible solution point which has the minimal per-setup cost variation.

Choosing the merge combination to get a better solution

Selecting the minimal per-setup cost is not enough to find the best solution. There can be cases, when the sum of per-setup cost variation of two products can be substitute for another product per-setup cost variation for a better solution. Figure 5 presents the per-setup cost variation of three products and the possibility of substitution.

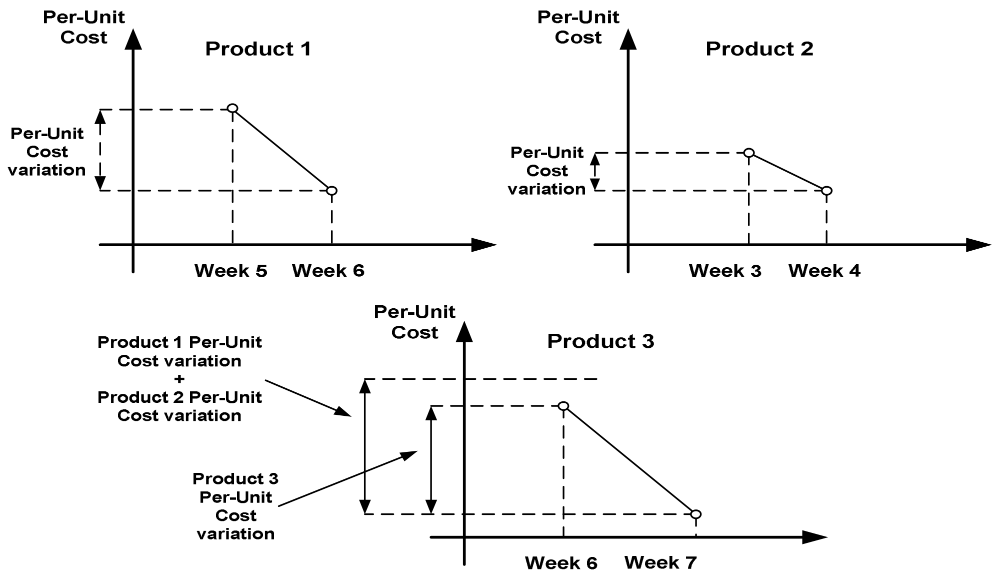

Figure 5. Comparison and substitution of per-setup cost increases

Explanation of figure 5: Because of the original computed solution does not fit in the capacity constraint condition; the number of jointly produced weeks are reduced by the help of the algorithm. According to the first step, the product having the minimal variation value of per-setup cost will be chosen. Be this the first product. Suppose that, the solution through reduction of jointly produced weeks from six to five still does not fit the capacity constraint. In case of one product, the substitution phase cannot be explained, so the algorithm runs on. In the second step, again a product will be chosen, which have the minimal variation value of per-setup cost. Be this the second product now. It is not sure that the reduction of jointly produced weeks of the two chosen product is optimal solution. Therefore we examine that the sum of variation increases of per-setup cost, gained from the reduction of the jointly produced weeks of the two chosen products, can be substituted for a smaller variation of per-setup cost. Figure 5 shows properly that the variation of per-setup cost of product three is smaller than the variations sum appeared at product one and two.

This means that the variations sum appeared at product one and two can be substitute for a reduction of jointly produced weeks at product three. Examine the case, if there are four products. In the next figure, the per-setup cost variations of product three and four can be seen in case of one week decreases of its jointly produced weeks.

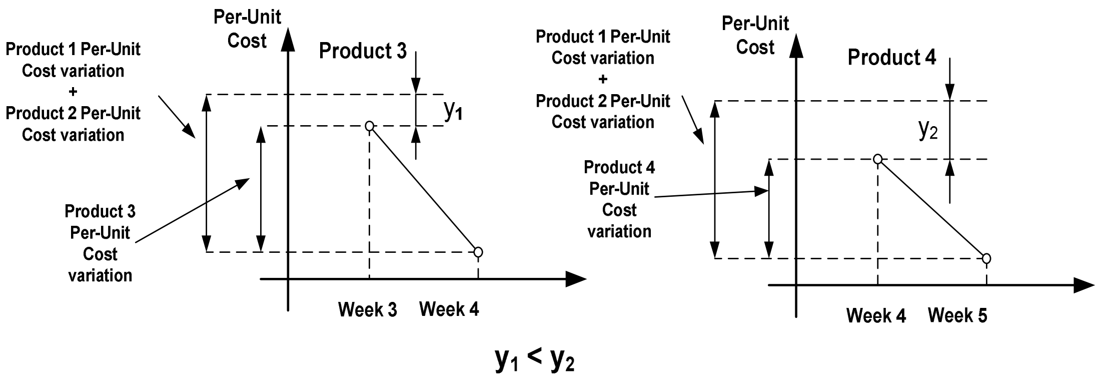

Figure 6. Substitution phase in case of more than three products

In case of more than three products, a question arises: Which product should be chosen for substitution? In Figure 6 can be seen that both of the variations of per-setup cost at product 3 and 4 are smaller than the variations sum appeared at the first two products.

In this case the product should be chosen, where the variation falls the farthest from the variations sum. The main reason of this is the following: if the result after the substitution does not fit the capacity constraint, the algorithm in the next step chooses again a product, which has the minimal per-setup cost variation. In case of this example, the first product will be this again. Suppose that the solution is optimal now. If product four is chosen, then the sum of per-setup cost variations is certain smaller as if product three would be chosen for substitution.

The substitution steps continuing the algorithm are as follows:

Step 4.: We examine, how many 'true values' are in the vector *B[i] (i =1...n)*. If it contains only one element, then there cannot be used the substitution, go to Step 2.
If it contains more than one element, then we summarize the variations of per-unit cost where *B[i]=true*, except the element has the *maxFSVK* index.

$$\textbf{Sum} = \sum_{i=1}^{n} P_i[opt_i - L[i]],\ if\ B[i] = true\ and\ i! = max\,FSVK\ .$$

Step 5.: Compare value of *Sum* variable with values of $P_i[opt_i - L[i]]$ *(i=1...n)*, where *B[i]=false*. We examine only the $Sum > P_i[opt_i - L[i]]$ cases, where the difference $(Sum - P_i[opt_i - L[i]])$ is the greatest.
If there is not such a product, jump to Step 2. Otherwise jump to Step 6.

Step 6.: Based on minIndex got it in the Step 5, we perform the real substitution:

$$L[minIndex] = L[minIndex] + 1,$$

$$B[minIndex] = true,$$

$$L[i] = 0, B[i] = false;\ Ha\ i! = minIndex,\ i = 1..n\,.$$

Jump to Step 2.

The result of the algorithm is a production "plan", where the sum of the producing quantity is optimal for the capacity constraint condition and the combination of week reductions is fill rate and cost-optimal.

4. CONCLUSION

In this paper we extend the previously elaborated and modified newsvendor model [5][6] with the condition of global capacity constraint. The extended model makes possible the determination of a cost-optimal stockpiling policy applying capacity constraint in case of optional number product and optional long production time horizon. Because of the individual approach of the new model, it can be used in practice effective in relation to other models in the literature.

The practical implementation of this inventory control method can be tested at the http://alpha.iit.uni-miskolc.hu/ICWeb/ web page.

5. ACKNOWLEDGEMENTS

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