MIM'07. IFAC workshop on manufacturing modelling, management and control. Budapest, Hungary, 2007. pp 123-126.

EVALUATING THE PROPER SERVICE LEVEL IN A COOPERATIVE SUPPLY CHAIN ENVIRONMENT

Péter Mileff, Károly Nehéz

University of Miskolc, Department of Information Engineering
Egyetem Road, Miskolc-Egyetemváros 3515, Hungary

Abstract: In the last decade, the business environment of companies in the field of discrete manufacturing has altered. The demand rate for mass products has remained at a high level but numerous new requirements have appeared on the market. The former, simple buying-selling (so-called „cool") relation has become more and more „warm". The life cycle of products is becoming ever shorter. Customer needs for stylish forms, new modern designs, special packaging or better product properties have greatly increased. Several innovative models have been developed to support inventory management decisions in this new environment. Classical inventory control models are not capable to handle such market models which have high demand fluctuation and other uncertainties any more. In this paper, we present a further development of the classical newsvendor model with multi-period extensions. We investigate relationships and cooperation level between the partners at the low, model level. We have determined an analytic solution method to handle these problems. The model helps the supplier to make proper decisions at controlling its inventory corresponding to the main goals of the company and the contract commitments.

Keywords: supply chain management, inventory control, extended newsvendor model, service level

1. INTRODUCTION

The business environment of companies in the field of mass production has altered in the last 15 years. The demand rate for mass products has remained at a high level but numerous new requirements have appeared on the market. The life cycle of products is becoming ever shorter. Customer needs for stylish forms, new modern designs, special packaging or better product properties have greatly increased. Generally, mass production companies assemble and bundle their products from components originating from their supplier companies.

Changes in the business environment influence engineering and logistic relations between companies and suppliers. The former, simple buying-selling (so-called "cool") relation has become much "warmer". This means that cooperative and collaborative methods and activities have become the main object in SCM development. The fast evolution of IT technology plays an important role in this process. In many respects, real-time, network-similar collaboration of independent, locally-separated companies is not realizable without an effective computer network information system.

The whole productive-marketing chain of mass production is fairly long. The customer demands appear in shopping centres, which generate orders to logistical centres. Logistical centres transmit these demands to end-product manufacturers. End-product manufacturers forward orders to dozens of suppliers. This process generates on-floor orders (internal orders), starts production of lots, and places orders for raw material from suppliers. These multi-stage distributed information, decision and physical (producing and transporting) supply chains, material- and information-transmitter chains have a unavoidable delay, which directly leads to delays and instabilities, back orders, and overstock and becomes a source of unusable loss. Developing complex, large, collaborative supply systems necessitates increased information technology support of both business and technical processes. Complex ERP systems and auxiliary SCM modules and standalone SCM applications are available on the market to support the above-mentioned planning, decision, executive and information processes.

MIM'07. IFAC workshop on manufacturing modelling, management and control. Budapest, Hungary, 2007. pp 123-126.

Relations of the marketing organizations, end-product manufacturers and supplier companies can be very complicated and various in practice. This motivates a wide examination of the available models and further investigation of effective decision supporting and planning methods.
In this paper we examine the possibility of supplier inventory policy in the case of non-deterministic demands. We assume that estimations concerning the future (forecasts) are solved; furthermore orders, acknowledgements, demands of delivery and the organization of the transport operation and synchronization of the planning process are also solved on the tactical level. We suppose that the supplier network on the strategy level is complete and bound by contract, and also that computerized communication conditions are available for the realization of business processes.
This paper presents an extended inventory control method based on the classical newsvendor concept. The model with handling more ordering periods jointly extension makes an effective and cost-optimal stockpiling policy possible for the supplier. The sufficient service level prescribed in the contracts plays an important part at the partner's cooperation. We investigate this problem at the low, model level. A new approach is presented, which helps tuning the service level in according to the contract requirements and the goals of the company.

2. THE EXTENDED NEWSVENDOR CONCEPT

The classic newsvendor model (Ayhan, et al., 2004) considers a type of problem that many decision makers (newsvendors) encounter in the business world. Facing uncertain demands for limited-useful-life products (such as mobile phones, fashionable goods etc.), a decision maker (newsvendor) needs to decide how many units of these goods to order for a single selling period. Intuitively, if she/he orders too many (surplus), this may cause unnecessary inventory cost. Thus, the cost will be too high. Whereas, if the decision maker orders too few (shortage), it will miss opportunities for additional profits because some customers have no chance to buy the goods. The optimal solution to this problem is characterized by a balance between the expected costs of shortage and surplus.
The model is certainly among the most important models in operations management. It is applied in a wide variety of areas: centralized and decentralized supply chain inventory management (e.g., Shang and Song 2003, Cachon, 2003), retail assortment planning (e.g., van Ryzin and Mahajan, 1999), international operations (e.g., Kouvelis and Gutierrez 1997), horizontal competition among firms facing stochastic demand (e.g., Lippman and McCardle, 1995), lead time competition (e.g., Li 1992), outsourcing and subcontracting decisions (e.g., Van Mieghem 1999), product and process redesign (Fisher and Raman 1996 and Lee 1996), and spot markets and inventory control (e.g., Lee and Whang 2002).
This model considers "short-time decision" horizon. Since setup cost appears in each ordering period (e.g.: weekly), real customized mass production inventory problems cannot be applied. The model does not fit the multi-period requirements (the quantity of demands should be produced jointly), production has been done in each period. Production experiences in practice shows clearly that choosing the proper number of the jointly produced periods is very important and affects the total cost of the company. The lower is this value the higher is the production initializing costs. It is not rare, whether these costs are higher than the whole production cost. The newsvendor model, own its periodicity, is an extreme case of this concept. The second extreme case is, whether too much periods are produced jointly. This time holding and working capital cost can attain high values. Therefore the optimal solution is between somewhere these two cases.

2.1 The model

Our goal was to develop a newsvendor based model, which can be applied effectively in multi-period decision environments. Based on the requirements of the mass production we formulated a new, extended cost function as follows:

$$\begin{aligned} K_{123.n}(q_{123.n}) &= c_f + c_v[q_{123.n} - x] + hE[q_{123.n} - D_1]^+ + \\ &+ hE[q_{123.n} - D_1 - D_2]^+ + \ldots + \\ &+ hE[q_{123.n} - D_1 - D_2 - \ldots - D_n]^+ + \\ &+ pE[D_1 - q_{123.n}]^+ + pE[(D_1 + D_2) - q_{123.n}]^+ + \ldots + \\ &+ pE[(D_1 + D_2 + \ldots + D_{n-1}) - q_{123.n}]^+ + \\ &+ pE\left[D_n + \left[\ldots + \left[D_2 + [D_1 - q_{123.n}]^-\right]^-\right]^-\right]^+ \end{aligned} \quad (1)$$

$$(q - D)^+ = max(q - D, 0), (D - q)^+ = max(D - q, 0).$$

Model parameters:
- c_f – fixed cost. This cost always exists when the production of a series is started.
- c_v – variable cost. This cost type expresses the production cost of one product.
- p – penalty cost. If there are fewer raw materials in the inventory than needed to satisfy the demands, this is the penalty cost of the unsatisfied orders.
- h – inventory and stock holding cost.
- D_i – this means the demand from the receiver in period i for the product, which is an optional probability variable.
- $E[D]$ – expected value of the D stochastic variable.
- $F(D)$ – cumulative distribution function of D.
- q – the product quantity in the inventory. The decision of the inventory control policy concerns the product quantity in the inventory

after the product decision. This parameter includes the initial inventory as well. If nothing is produced, then this quantity is equal to the initial quantity, i.e. concerning the existing inventory. $q_{123\ldots n} = q_1 + q_2 + \ldots + q_n$.

x – initial inventory. We assume that the supplier possesses x products in the inventory at the beginning of the demand of the delivery period.

This extended cost function describes the total cost of the supplier for an optional length (n period) production horizon. It is easy to see that the model follows the newsvendor concept, except the periodicity production. This concept and the solution give an extended answer for one of the basic question of inventory control problems: "How much to produce?". The main importance of the solution is, it can be calculated in closed form with the following equation:

$$F_{123.n}(q_{123.n}*)=\frac{p-c_v-hF_1(q_{123.n}*)-hF_{12}(q_{123.n}*)}{p+h}+ \\ +\frac{-hF_{123}(q_{123.n}*)-\ldots-hF_{123.n-1}(q_{123.n}*)}{p+h}, \quad (2)$$

where $F()$ represents the joint distribution function in compliance with the number of periods drawn together. The type of these distributions can be optional for all the periods, although normal distribution is used most often in practice. The reason of this that calculating the joint distribution function of the sum of different probability random variables requires very complex mathematical skills. The $q_{123\ldots n}{}^*$ - which satisfies the equation - expresses that the finished goods must be in the inventory at the time when customer demand appears with regard to n periods.

Equation (2) can be solved by using numerical methods but in special cases some simplification can be applied.

In practical calculations, values of $hF_1(q_{123\ldots n}), hF_{12}(q_{123\ldots n}), hF_{123}(q_{123\ldots n}), \ldots, hF_{123\ldots n-2}(q_{123\ldots n})$ can be approximated by 1. Note that: assuming that $F(x)$ *is a* cumulative distribution function of a uniform distribution. If argument x is greater than the maximum value of the given uniform distribution, then $F(x)$ always gives 1 by definition. This way equation (2) becomes a simplified form as follows:

$$F_{123.n}(q_{123.n}*)=\frac{p-c_v-(n-2)\cdot h-hF_{123.n-1}(q_{123.n}*)}{h+p}. \quad (3)$$

The numerator can be a negative value in that case, when sum of holding costs during the periods is greater than a certain limit and naturally there is no optimal solution. This time the number of periods have to be reduced, because it is cheaper if the supplier does not produce anything.

Therefore it is easy to see that the penalty parameter plays an extremely important role in determining the optimal stockpiling policy in function of the allowable back-orders. In the following we investigate this problem in details. Extending the classical model to handle more logistic periods is necessary, but not a sufficient condition. Only the determination of the optimal number of jointly produced periods minimizes the total cost of the company. Paper (Mileff and Nehéz, 2006) consider this problem in details.

3. THE CRITICAL (s) INVENTORY LEVEL

It is easy to see that if there are no available products in stock to satisfy the demand, then necessarily a production cycle should not be started, because it carries fixed costs which make the production of a small volume expensive. Consequently it is conceivable that there definitely exists a critical amount, which is smaller than the optimal amount, but by choosing this quantity it is more profitable to sustain the risk of the back-order. The name of that point where the cost of the decision about producing or non-producing (as we would rather undertake the risk of the back-order) is equal is the critical inventory level (s) (Hayriye, 2004). The critical level is probably smaller than the long-term cost-optimal inventory level. If the products in the inventory are less than this, only then is it profitable to increase the stock in hand to the optimal level. In literature this approach of inventory control is known as (S,s) and (s,q) policy.

In this paper we show how the critical inventory level can be calculated using our extended newsvendor model.

3.1 The solution method

Let introduce with $L_{123\ldots n}(q_{123\ldots n})$ notation the sum of holding and penalty costs for n periods. Then

$$\begin{aligned} &Lq_{123\ldots n}(q_{123\ldots n}) = \\ &= hE[q_{123\ldots n} - D_1]^+ + hE[q_{123\ldots n} - D_1 - D_2]^+ \quad (4) \\ &+ \ldots + hE[q_{123\ldots n} - D_1 - D_2 - \ldots - D_n]^+ + \\ &+ pE[(D_1 + D_2 + \ldots + D_n) - q_{123\ldots n}]^+ . \end{aligned}$$

Assume that initial inventory level is less than the optimal, $x < q^*_{123\ldots n}$. When we raise this level to the optimal, the cost will be $K_{123\ldots n}(q^*_{123\ldots n}) = c_f + c_v(q^*_{123\ldots n} - x) + L(q^*_{123\ldots n})$. If the supplier does not produce any product, only with the initial inventory should calculate ($L_{123\ldots n}(x)$).

The objective is to find the $s_{123\ldots n}$ critical amount, where production and non-production costs are equal. If $L_{123\ldots n}(x) < K_{123\ldots n}(q^*_{123\ldots n})$, then the supplier does not need to produce, because setup and production costs increase its total cost. To find the critical level, the following equation should be sold for $s_{123\ldots n}$.

$$L(s_{123\ldots n})+c_v s_{123\ldots n}=c_f+c_v q^*_{123.n}+L(q^*_{123.n}) \qquad (5)$$

The solution $s_{123\ldots n}$ means that if the inventory of the supplier is less than the critical level, then $q^*_{123\ldots n}$-x amount product should produce.
The critical inventory level approach can extend the stockpiling policy of the supplier, but using it is a collaborative question which depends on the contractual relationship among the partners. When contracts do not allow any back-order, this approach cannot be applied.

4. PENALTY AS CONTROL PARAMETER

Applying the model at the practice requires using the model parameters properly. This means that it is necessary to determine these parameters in a way, because the solution according to these variables will only be optimal.
The penalty parameter has a special role in the model, because cooperation interests appear through its value. In this approach the penalty value like "control parameter" affects basically the supplier's policy. Generally in practice the number of back-orders is given for a production horizon. At this point suppliers should decide what kind of penalty value to be used at the determination of the stockpiling policy. The estimation of the other parameter values is relatively less complicated.
Periodic models in literature, which allow back-orders, investigate this problem particularly from theoretical side, because generally there is no reference to the evaluation of the penalty value appearing in the objective function.
However the practicability of the model requires choosing the proper value of this parameter considering the long-term contracts. So the penalty parameter can fill the "control" parameter part at the cooperation of the partners.
We present a new approach to handle this problem in our extended newsvendor model. This method can determine the exact value of this parameter in closed, analytic form in function of the given service level.

4.1 Calculating the proper service level

To determine the penalty value, let start with the equation of the expected back-order:

$$v_{123\ldots n}=E\left(D_{123\ldots n}-q_{123\ldots n}\right)^+, \qquad (6)$$

where $v_{123\ldots n}$ represents number expected back-orders for n number of periods. Our objective is to control the back-orders, so we assume that $v_{123\ldots n}$ comes as model parameter from the higher levels. Assume that at least zero back-order occurs ($v \geq 0$), so *max* operator can be eliminated. Moreover we assume that the supplier produces optimal quantity. Replace $q_{123\ldots n}$ in Equation 6 with the optimal q_{123n}* solution (Equation 2). Then we get the following formula:

$$Y:=\frac{p-c_v-hF_1(q_{123.n})-hF_{12}(q_{123.n})-\ldots-hF_{123.n-1}(q_{123.n})}{p+h}$$

$$v_{123.n}=E\left[D_{123.n}-F_{123.n}^{-1}(Y)\right]^+, \qquad (7)$$

where $F^{-1}()$ means the inverse distribution function. The objective is to determine the p value from Equation 6. It is easy to see that $q_{123\ldots n}$ appears in the equation. To eliminate this we use the known formula of the expected back-order:

$$q_{123..n}=E(D_{123..n})-v_{123..n} \qquad (8)$$

Substituting this formula to Equation 7 makes possible to determine a closed formula to the penalty parameter as follows:

$$p=\frac{c_v+hF_1(E(D_{123n})-v_{123n})+hF_{12}(E(D_{123n})-v_{123n})}{F_{123n}(E(D_{123n})-v_{123n})-1}+ \\ +\ldots+\frac{hF_{123n}(E(D_{123n})-v_{123n})}{F_{123n}(E(D_{123n})-v_{123n})-1}. \qquad (9)$$

The formula determines exactly the value of penalty cost in function of supplier cost parameters, the type of distribution function and the number of allowed back-orders. Applying this formula the supplier can ensure the specified service level.

4.2 Property of the solution method

Because of the stochastic property of solution (9), the result can be interpreted in two different ways. If we use the long term expected value of the demands, the solution can be explained only to the same long distance. At this time $v_{123\ldots n}$ means the average number of back-orders. In that case, when the length of the time horizon is short (e.g.: one period) and the number of back-orders is minimized in contracts, the method cannot use directly. To apply the method for short horizon, the maximum value of the demand should be used to solve the equation. In this case $v_{123\ldots n}$ means the maximum value of the allowable back-orders. Let see the accuracy of these approaches through a little example with two production periods. Assume that c_v = 5\$, and h = 2\$. We use normal distribution to model the demands as forecast values.
To make this example simpler, we assume that this value is identical in all periods ($\mu = 15$ products). Let the reliability of the forecast $\sigma = 3$. We use the second interpretation of the model, so the maximum value of the demand should be determined. In case of $\sigma = 3$ the demand maximum will be 24 for one

period, which has 99.99% confidential level. Let v_{12} = 2 products, which means that two back-orders are allowed maximum during the two period.

$$p = -\frac{F_{12}(E(D_{12}) - v_{12})h + c_v + hF_1(E(D_{12}) - v_{12})}{F_{12}(E(D_{12}) - v_{12}) - 1} =$$

$$= -\frac{0.9967 * 2 + 5 + 2}{0.9967 - 1} = 1825.49\,\$.$$

Checking the p value:

$$q_{12} = F_{12}^{-1}\left(\frac{1825.49 - 5 - 2}{1825.49 + 2}\right) \approx 46 \; .$$

It is easy to see when the supplier produces 46 products for two periods jointly and demands are at the maximum level in all the periods (24*2 = 48 products), then the supplier can have only $v_{12} = 2$ back-orders.

Observing the solution approach deeply it can be seen that applying the method requires to know the length of the production time horizon. In practice the supplier knows neither the number of jointly produced periods, nor the value of penalty cost.

In this case determining the optimal stockpiling policy is a cyclic process using the per-unit cost model discussed in paper (Mileff and Nehéz, 2006).

5. ACKNOWLEDGEMENTS

The research and development summarized in this paper has been carried out by the Production Information Engineering and Research Team (PIERT) established at the Department of Information Engineering. The research is supported by the Hungarian Academy of Sciences and the Hungarian Government with the NKFP VITAL Grant. The financial support of the research by the aforementioned sources is gratefully acknowledged.

6. CONCLUSIONS

In the present paper we formulated a new inventory control model following and extending the newsvendor concept and based on the requirements of the customized mass production. The main advantage of the proposed model is that the multi-period problems can be solved analytically. It makes possible for the supplier to answer the "what if" type investigations and quick decisions. We showed how critical inventory level approach can be applied at the extended model to make the stockpiling policy more effective for the supplier.

To determine the appropriate stockpiling policy in compliance of the given service level prescribed in the contracts we presented a new approach. We point that the penalty value can be a control parameter between the cooperative partners. With the help of the presented analytic method and determining the exact value of the penalty cost, the supplier can ensure the proper production quantity and the service level.

REFERENCES

Ayhan, H., Dai, J., Foley, R. D. and Wu, Joe (2004). Newsvendor Notes. *ISyE 3232 Stochastic Manufacturing & Service Systems*. pp. 67-78.

Cachon, G. P. (2003). Supply Chain Coordination with Contracts, In de Kok, A. G., Graves, S. C. (eds): Supply Chain Management: Design, Coordination and Cooperation. *Handbooks in Op. Res. and Man. Sci.,* **11***, Elsevier*, pp. 229-339.

Lee, H., Whang, S. (2002). The impact of the secondary market on the supply chain. *Management Science*. **Volume 48**, pp. 719-731.

Lippman S, and Mccardle K. (1997). The Competitive Newsboy. *Operations Research,* **Volume 45**, pp. 54-65.

Mieghem, J.A. (1999). Coordinating Investment, Production and Subcontracting. *Management Science*. **Volume 45, Number 7**, pp. 954-971.

Mileff, P. and Nehéz K. (2006). An Extended Newsvendor Model for Customized Mass Production, *AMO - Advanced Modelling and Optimization*, *Electronic International Journal*, **Volume 8, Number 2**, pp. 169-18.

Ryzin, G.J. and Mahajan S. (1999). On the Relationship Between Inventory Cost and Variety Benefits in Retail Assortments. *Management Science*, **45**, pp. 1496-1509.

Shang, K. H., Song, JS and Zipkin, P. H. (2006). Coordination Mechanisms in Decentralized Serial Inventory Systems with Batch Ordering. *Working Paper, Fuqua School of Business, Duke University*.