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AN EXTENDED NEWSVENDOR MODEL FOR SOLVING CAPACITY CONSTRAINT PROBLEMS IN A MULTI-ITEM, MULTI-PERIOD ENVIROMENT

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Abstract. In the last few years, the effective management of the inventory control problems became ever more critical problem of the supplier companies. In this paper, on the basis of demands of a major Hungarian mass production company, an extension of an analytical inventory control model is presented considering the condition of global capacity constraint. This model was elaborated in our previous paper [6] and models the one customer - one supplier relation. Our aim is to determine an optimal holding-production policy of the supplier, which makes a cost-optimal stockpiling policy possible for an optional long production time horizon. We show, that building on our former results, the global capacity constraint satisfying policy can be determined by the help of a new heuristic method.

Keywords: stockpiling policy, extended newsvendor model, global capacity constraint

1. Introduction

In the last 15 years, the business environment of companies in the field of mass production has been altered. The demand rate for mass products has remained at a high level but numerous new requirements have appeared on the market. Changes in the business environment influence engineering and logistic relations between companies and suppliers. The former, simple buying-selling (so-called “cool”) relation has become much “warmer”. This means that cooperative and collaborative methods and activities have become the main object in SCM development. Relations of the marketing organizations, end-product manufacturers and supplier companies can be very complicated and various in practice. This

motivates a wide examination of the available models and further investigation of effective decision supporting and planning methods.
In the literature we can meet the wide scale of stockpiling models [6]. In the later we deal with one of the most known stochastic method, so-called "newsvendor model". The model is certainly among the most important models in the field of operations management. It is applied in a wide variety of stockpiling problems. In this paper in compliance with this, an extended newsvendor model [7] will be examined on the basis of former results in a multi-product and capacity constraint case.
The properties of our extended newsvendor model that the inventory control problem of an optional length production horizon can be solved analytically opened new opportunities to model multi-product capacity constraint problems. Capacity constraint problem appears almost all larger or smaller commercial firms. These firms produce often several products in compliance of customer demands. In case of dynamically and stochastically changing demands, often the capacity problem can be appeared. The question is always the same: how much should be produced? Of course the question is very simple, but the solution is NP-hard.
The model conditions are fully identical with the conditions presented in [5][6] publications. The greater part of the models in the literature solves the problem applying the tool of the dynamic programming or some kind of searching method (soft-computing). Because of the large searching space, these solutions require extremely long computation time in case of many product number and long production time horizon. During our research we investigated the capacity constraint problem from one product one period to more products and more periods. This paper presents only some of these methods.

1.1. An Extended Newsvendor Model

The classical newsvendor model cannot be applied properly to solve the tasks appearing at the customized mass production. The reason of this is the high value of the setup cost and it cannot handle multi-period problems, where customer demand can vary stochasticly. During our research we had bean developed a new inventory controll method, which gives the optimal solution for the problem in an analitic way, and assures efficient stockpiling for the supplier.
Summarizing the main characteristic of the model the objective function can be formulated as follows:

$$\begin{aligned} K_{123\ldots n}(q_{123\ldots n}) &= c_f + c_v\left[q_{123\ldots n} - I\right] + hE\left[q_{123\ldots n} - D_1\right]^+ + hE\left[q_{123\ldots n} - D_1 - D_2\right]^+ + \ldots + \\ &+ hE\left[q_{123\ldots n} - D_1 - D_2 - \ldots - D_n\right]^+ + pE\left[D_1 - q_{123\ldots n}\right]^+ + pE\left[\left(D_1 + D_2\right) - q_{123\ldots n}\right]^+ + \ldots + \\ &+ pE\left[\left(D_1 + D_2 + \ldots + D_{n-1}\right) - q_{123\ldots n}\right]^+ + pE\left[D_n + \left[\ldots + \left[D_2 + \left[D_1 - q_{123\ldots n}\right]^-\right]^-\right]^-\right]^+, \end{aligned} \quad (1.1)$$

where the individual parameters are the following:

cf – fixed cost. This cost always exists when the production of a series is started. [Ft / production]

c_v – variable cost. This cost type expresses the production cost of one product. [Ft / product]

p – penalty cost (or back order cost). If there is less raw material in the inventory than needed to satisfy the demands, this is the penalty cost of the unsatisfied orders. [Ft / product]

h – inventory and stock holding cost. [Ft / product]

D – this means the demand from the receiver for the product, which is an optional probability variable. [number / period]

E[D] – expected value of the *D* stochastic variable.

q – the product quantity in the inventory. The decision of the inventory control policy concerns the product quantity in the inventory after the product decision. This parameter includes the initial inventory as well. If nothing is produced, then this quantity is equal to the initial quantity, i.e. concerning the existing inventory.

I – initial inventory level. We assume that the supplier possesses *I* products in the inventory at the beginning of the demand of the delivery period.

The new method is robust and adequatly elegant (detailed in paper [6][7]), because the solution is independent from the type of the distributed function:

$$F_{123.n}(q_{123.n}{}^*) = \frac{p - c_v - hF_1(q_{123.n}) - hF_{12}(q_{123.n}) - hF_{123}(q_{123.n}) - \ldots - hF_{123.n-1}(q_{123.n})}{h+p}, \quad (1.2)$$

where $F()$ represents the joint distribution function in compliance with the number of periods drawn together. The $q_{123\ldots n}{}^*$ - which satisfies the equation - expresses how many finished goods should be in the inventory at the time when customer demand appears with regard to *n* periods. Of course the critical inventory level mentioned first by Herbert Scarf [1] for one-period production can be applied in the case of joint production for *n* numbers of production cycles too. However this is not discussed in this paper.

2. The One Product, Multi-Period Model

Introducing the global capacity constraint for the multi-period extended newsvendor model we consider the characteristic of the model, the period based policy. Accordingly two type of optimization methods can be differed: the *service level based policy* and *cost based policy*. Of course these policies are in contrast with each other. Only one of them can be preveal in the stockpiling.

2.1. Cost Based Policy

This approach models the type of a supplier, which is directly in relation to the market. In case of this type of policy, the main objective is to minimize the costs. It should be decided that how many back-orders can be occurred during the time horizon. So the penalty cost is determined by the supplier itself [7]. Because in case of cost based policy keeping the service level is not the main objective, the solution at the optimization of the production costs can be *the reduction of the number of setups* or *taking the risk of the penalty*.

The reduction of the number of jointly produced periods is not definitely the best solution, the minimal cost case. It is possible that taking the penalty cost is cheaper for the supplier.

Theorem 1: if the sum of the penalty cost appearing at producing the quantity accordingly to the capacity contraint and the holding cost of quantity of the truncated period storing from the beginning of the time horizon, is less than the cost of prepearing a new setup, then taking the risk of the penalty is the proper policy.

We prove this theorem with the following explanation. Denote the cost value of the occurred back-orders with variable *b*. The next equation helps to decide what policy should be applied.

$$\text{if} \begin{cases} b + \sum_{i=1}^{a} h \cdot (C - q_a) \le c_f, \text{ then taking the risk}, \\ b + \sum_{i=1}^{a} h \cdot (C - q_a) > c_f, \text{ then reducing the periods}. \end{cases}, \tag{2.1}$$

where h is a cummulative holding cost per products and q_a< C is the quantity in compliance with the number of jointly produced periods. Variable a means the period number, which q_a value is even smaller than the C capacity constraint. Then

$$h \cdot a \cdot (C - q_a) = \sum_{i=1}^{a} h \cdot (C - q_a) \tag{2.2}$$

represents the holding cost, which appears as difference of the quantity of the capacity constraint and the quantity of the reduced jointly produced periods. The meaning of the formula: In the case, when the value of $b + h \cdot a \cdot (C - q_a)$ is cheaper than a new setup cost (c_f), the cost will be minimal, when the supplier chooses to take the penalty and produces the quantity of the capacity constraint. Otherwise reducing the number of the jointly produced periods is a good policy. Figure 1 shows the method of the cost based policy properly.

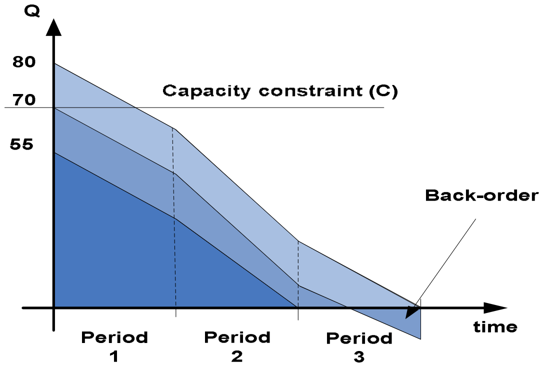

Figure 1. Applying capacity constraint in case of cost base policy

2.1. Service Level based Policy

The main objective when choosing this policy is to assure the predeterminded *Service Level* continually. This level is determined in compliance with the objectivel of the company. Applying capacity constraint means that the number of unsatisfied orders should be less than the strict service level. Unkeeping this important rule takes the risk of the relationship of the customer and the supplier.
The reduction of the jointly produced periods gives the solution of this problem. If the optimal qantity calculated with the extended newsvendor model exceeds the value of the capacity constraint, then the predetermined service level can be kept only the way, that we reduce the number jointly produces periods until the quantity according to the reduced period satisfies the capacity condition. The reason of this solution the per unit cost variation curve detailed in paper [7].

3. The Multi-Product, Multi-Period Model

Solving capacity constraint problems in case of this model is the most complicated. In the literature the ABC method is applied widely to solve the problem. In this case, conclusions can be made based on the prepared Pareto diagram about the "significance" distribution of the elements of a product set [4]. But the method does not answer properly the questions appearing at the calculation of the optimal stockpiling quantities.

In the following we present a new heuristic method to solve multi-product, multi-period and service level based capacity constraint optimization problems. We assume a global capacity constraint, which means that products share in one common production capacity and assume that the decisions of the inventory control policy regard to long time horizon.

In this presented solution we prefer Service Level based policy. The objective is to determine the reduced number of jointly produced periods per product in the way that the sum of the total amounts satisfies the capacity constraint condition.

The main idea behind our new heuristic solution is the specific property of the per-unit cost of the products. Figure 2 shows the variation of the per-unit cost of a product in function of jointly produced periods.

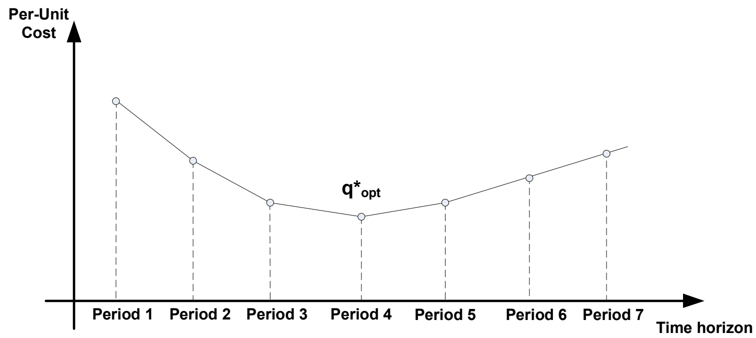

Figure 2. Per-unit cost variation in case of seven periods length time horizon

Each product has a similar per-unit curve [7]. If the sum of the quantities of n number of products is greater than the value of capacity constraint, then the solution should be modified. If

$$\sum_{i=1}^{n} u^i \cdot (q_{opt_j}^{i*} - I^i) \leq C \text{, the solution is optimal.} \tag{3.1}$$

In the equation i (i=1,…,n) means the number of products, $q_{opt_j}^{i*}$ is the optimal quantity of the product i in case of opt_j number of jointly produced periods and u^i represents the capacity usage of the product. I^i denotes the initial inventory of product i. The question is, which setup number of product should be modified to have the solution minimal cost?

Start from examination of the per-unit cost curve. The next figure shows the variation of the per-unit cost for 4 periods based on the modification of Figure 2.

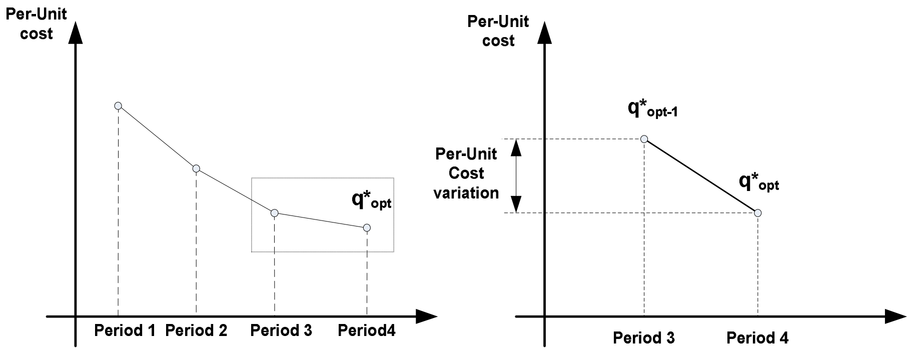

Figure 3. Increase of per-unit cost in function of jointly produced weeks

From Figure 3 it is easy to see, if the optimal number (4) of jointly produced periods is decreased to three periods, the value of the per-unit cost will be certainly increased. Based on this observation:

Theorem 2: the capacity constraint suitable solution is optimal, when the sum of the per-unit cost increases arisen from reducing the number of jointly produced periods at the products, is minimal.

Denote FKV_i the sum of performed per-unit cost variations at product i. Then:

$$\sum_{i=1}^{n} FKV_i \longrightarrow \min . \qquad (3.2)$$

Theorem 2 helps finding the optimal solution, but an essential searching method is necessary, which can calculate the sum of per-unit cost variations fast in a multi-product, multi-period environment. In the following we present a new, and suitable algorithm.

3.1 The Algorithm and the Other Parts of the Method

The basic idea behind the algorithm is the existence of the optimal solution per product without the capacity constraint condition. The objective of the method is to move on the searching space along the minimal per-unit cost variations, because we already mentioned before that the optimal solution has minimal sum of per-unit variation costs. The algorithm can be divided three main parts: (1) starting the method and checking the capacity constraint condition (2) selecting optimum possible modifications (3) choosing the merge combination to get a better solution.

During the solution search these steps are continually repeated util the optimal supplier policy will be found in compliance the capacity constraint condition.

3.1.1 Start and Capacity constraint condition test

The first step of the method is to determine the optimal number of jointly produced periods based on the introduced per-unit cost model [8]. This operation is performed only once during the running, at the start. After that it should be investigated that is there any products which production can be "shift" forward. This can be performed by comparing the quantities in the inventory and the optimal quantities according to the jointly produced periods. Regarding to the first period:

$$c_v\left(q_j^{i*} - I_1^i\right) \le 0 \, , \, j = 1,2,\ldots, m \, , i = 1,2,\ldots, n \tag{3.3}$$

If a product can be found where this equation is fulfilled during the calculation of the per-unit costs, then the production of this product can be shifted forward along the time. After this, these products do not take part in the further steps.
The next step is the evaluation of the following capacity constraint condition.

$$\sum_{j=1}^{n} u^i q^i_{opt_j - Li} - I^i \le C \, . \tag{3.4}$$

If the condition is satisfied, then the solution is optimal. The equation has a new member L_i which modifies the optimal number of jointly produced periods. L_i represents the solution vector, which will store the reduced jointly produced periods of the products after the iteration steps of the algorithm. At the beginning of the iteration, this is a zero vector. If the equation is not satisfied next steps follows.

3.1.2 Choosing the optimum possible modifications

If the solution at the first step or in a previous iteration do not satisfies the capacity constraint, then the modification of the solution is required. At the second step of the algorithm those products will be chosen, which can be suitable at the determination of the optimal solution. Choosing the optimum possible modification means always the product, which per-unit cost variation is minimal. To determine this product the following steps must be performed: the formerly calculated optimal number of jointly produced periods is reduced virtually with one period accordingly to the actual modifications (L_i). This means at product *i*: $opt_j - (L_i + 1)$.

Before and after the reduction, the per-unit cost and then the per-unit cost variation can be calculated.
Only one product having the minimal per-unit cost variation will be chosen during this iteration step. If product *i* have been chosen, then the *i.* element of the solution vector will be increased: $L_i = L_i + 1$. The following figure shows this reduction method properly.

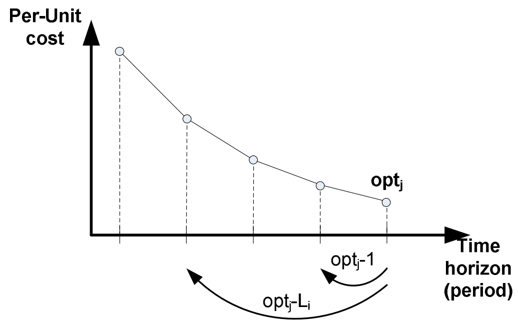

Figure 4. Reduction of the jointly produced periods in function of the solution vector

The product with the maximum per-unit cost variation will also be chosen if merging was performed in the previous iteration. This choice constitutes the base of the last step of the algorithm, which forbids the conformation of iteration loops (detailed at step 3).

3.1.3 Choosing the merge combination to get a better solution

Selecting the minimal per-setup cost is not enough to find the best solution. There can be cases, when the sum of per-setup cost variation of two products can be substitute for an another product per-setup cost variation for a better solution. Figure 5 presents the per-setup cost variation of three products and the possibility of substitution.

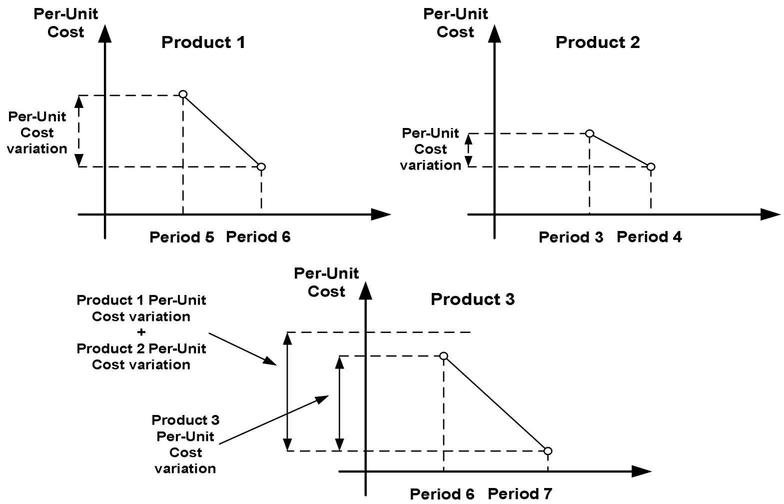

Figure 5. Comparison and substitution of per-setup cost increases

Explanation of figure 5: because of the original computed solution does not fit to the capacity constraint condition, the number of jointly produced periods is reduced by help of the algorithm. According to the first step of the algorithm, the product having the minimal variation value of per-unit cost will be chosen. Let it be the first product. Suppose that, the solution through reduction of jointly produced periods from six to five still does not fit the capacity constraint. In case of one product, the substitution phase cannot be explained, so the algorithm runs on. In the second step, again a product will be chosen, which have the minimal variation value of per-unit cost. Let it be the second product now.
It is not sure that the reduction of jointly produced periods of the two chosen product is the optimal solution. Therefore we should examine that the sum of variation increases of per-unit cost, gained from the reduction of the jointly produced periods of the two chosen products, can be substituted for a smaller variation of per-unit cost. Figure 5 shows properly that the variation of per-unit cost of product three is smaller than the variations sum appeared at product one and two.
This means that the variations sum appeared at product one and two can be substitute for a reduction of jointly produced periods at product three. Examine the case, if there are four products. In the next figure, the per-unit cost variations of product three and four can be seen in case of one period decreases of its jointly produced periods.

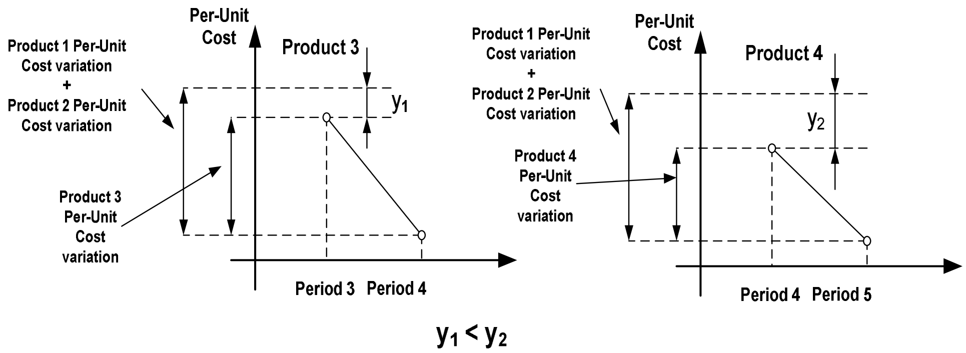

Figure 6. Substitution phase in case of more than three products

In case of more than three products, a question arises: which product should be chosen for substitution. In figure 6 can be seen that both of the variations of per-unit cost at product three and four are smaller than the variations sum appeared at the first two products.
In this case the product should be chosen, where the variation falls the farthest from the variations sum. The main reason of this is the following: if the result after the substitution does not fit the capacity constraint, the algorithm in the next step

chooses again a product, which has the minimal per-unit cost variation. In this example, the first product will be this again. Suppose that the solution is optimal now. If product four is chosen, then the sum of per-unit cost variations is certain smaller as if product three would be chosen for substitution.
During the substitution process we use the product with maximum per-unit cost variation value found at the second step. This value and product constitutes the base of reference in the investigation of the per-unit cost variations. The possibility of merging will be examined according to this value, because it cannot be the chosen product.
If the substitution performed successfully it is necessary to prepare the next iteration. The first step is to modify the solution vector. The value in the vector belonging to the selected product should set to zero. This ensures that the algorithm can move on the search space along the minimal per-unit cost variations.
After this process the next iteration comes until the solution does not fit the capacity constraint condition.
Calculations in practical show clearly that finding the optimal solution do not need a lot of iteration step. In case of a product, the optimal number of jointly produced periods is about 7-8 periods. The analytic solution of the extended newsvendor model assures a high performance calculation in an optional multi-product, multi-period environment in case of long time horizon.

4. Conclusion

In this paper we extended the previously elaborated and modified newsvendor model [6][7] with the condition of global capacity constraint. Based on the periodic characteristic of the model, two problem groups were differed and presented. For the most complex, multi-period, multi-product case a new heuristic method was elaborated. This model makes possible the determination of a cost-optimal stockpiling policy applying capacity constraint in case of optional number product and optional long production time horizon. Because of the individual approach of the new model, it can be used effectively in practice in relation to other models in the literature.

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REFERENCES

[1] HAYRIYE, A., JIM, D.,FOLEY, R. D., JOE, W.: *Newsvendor Notes*, ISyE 3232 Stochastic Manufacturing & Service Systems, 2004.

[2] CACHON, G. P.: *Competitive Supply Chain Inventory Management, Quantitative Models for Supply Chain Management*, International Series in Operations Research & Management Science, 17), Chapter 5, 2003.

[3] CACHON, G. P.: *Supply Chain Coordination with Contracts*. In de Kok, A. G., Graves, S. C. (eds): Supply Chain Management: Design, Coordination and Cooperation. Handbooks in Op. Res. and Man. Sci., 11, Elsevier, 2003, pp. 229-339.

[4] TAYLOR, A. D.: *Supply Chains A Managers Guide*, Addison Wesley, 2003.

[5] HANS-JOACHIM, G., CHIKÁN, A.: *The Origins of Dynamic Inventory Modelling under Uncertainty*, International Journal of Production Economics Volume 71, Issues 1-3, 1999, pp. 25-38.

[6] MILEFF, P.: *Kiterjesztett újságárus modell alkalmazása az igény szerinti tömeggyártás készletgazdálkodási problémáiban*, PhD thesis at Hatvany József Informatikai Tudományok Doktori Iskola, 2008.

[7] MILEFF, P., NEHEZ, K.: *An Extended Newsvendor Model for Customized Mass Production*, AOM - Advanced modelling and Optimization. Electronic International Journal, Volume 8, Number 2, 2006, pp 169-186.

[8] MILEFF, P., NEHÉZ, K.: *A new inventory control method for supply chain management*, UMTIK-2006, 12th International Conference on Machine Design and Production, Istanbul – Turkey, 2006, pp. 393-409.

[9] BRAHIMI, N., DAUZERE-PERES, S., NAJID, N. M., NORDLI, A.: *Single Item Lot Sizing Problems*, European Journal of Operational Research, 168, 2006, pp. 1-16.

[10] LEE, C. C., CHU, W. H. J.: *Who Should Control Inventory in a Supply Chain?*, European Journal of Operational Research, 164, 2005, pp. 158-172.

[11] MILEFF, P., NEHEZ, K.: *Applying Analytical Methods in Inventory Control Problems*, Proceedings of microCAD 20th International Scientific Conference, Hungary, 2006, pp. 128-135.

[12] JULIEN, B., DAVID, S.: *The Logic of Logistics: Theory, Algorithms, and Applications for Logistics Management*, Springer PLACE of publication, Chapter 8-9, 1997.

[13] ARROW, K.J., HARRIS, T., MARSCHAK, J.: *Optimal inventory policy*, Econometrica 19: 250 – 272, 1951.

[14] KARLIN, S.: *The structure of dynamic programing models*, Naval Research Logistics Quarterly 2: 285 – 294, 1955.

[15] MILNE, A.: *The mathematical theory of inventory and production: The Stanford Studies after 36 years*, In Workshop, August 1994, Lake Balaton. ISIR, Budapest, 1996, 59 - 77.

[16] VON NEUMANN, J. AND MORGENSTERN, O.: *Theory of Games and Economic Behavior*, Princeton University Press, 1944.

[17] DVORETZKY, A., KIEFER, J., WOLFOWITZ, J.: *On the optimal character of the (s; S) policy in inventory theory*, 1953, Econometrica 21: 586 - 596.