Calculating the conclusion by approximating the vague points of the fuzzy rulebase

If all the vague environments of the antecedent and consequent universes of the fuzzy rulebase are exist, all the primary fuzzy sets (linguistic terms) used in the fuzzy rulebase can be characterised by points in their vague environment. So the fuzzy rules (build on the primary fuzzy sets) can be characterised by points in the vague environment of the fuzzy rulebase too.
This case the approximate fuzzy reasoning can be handled as a classical interpolation task.

For example we can adopt the method of **Lagrange interpolation** for vague environment.

The original formula is the following:

$$Y(x) = \frac{(x - x_2)\cdot(x - x_3)\cdot\ldots\cdot(x - x_n)}{(x_1 - x_2)\cdot(x_1 - x_3)\cdot\ldots\cdot(x_1 - x_n)}\cdot y_1 + \frac{(x - x_1)\cdot(x - x_3)\cdot\ldots\cdot(x - x_n)}{(x_2 - x_1)\cdot(x_2 - x_3)\cdot\ldots\cdot(x_2 - x_n)}\cdot y_2 + \ldots$$

$$\ldots + \frac{(x - x_1)\cdot(x - x_2)\cdot\ldots\cdot(x - x_{n-1})}{(x_n - x_1)\cdot(x_n - x_2)\cdot\ldots\cdot(x_n - x_{n-1})}\cdot y_n$$

where Y(x) is the Lagrange interpolation of the n points (x_1,y_1), (x_2,y_2),..., (x_n,y_n).

Using the **concept of vague distances** in the case of one dimensional antecedent universes we get:

$$\mathrm{dist}(y_0, y) = \frac{\mathrm{dist}(a_2, x)\cdot\mathrm{dist}(a_3, x)\cdot\ldots\cdot\mathrm{dist}(a_r, x)}{\mathrm{dist}(a_2, a_1)\cdot\mathrm{dist}(a_3, a_1)\cdot\ldots\cdot\mathrm{dist}(a_r, a_1)}\cdot\mathrm{dist}(y_0, b_1) + \ldots$$

$$\ldots + \frac{\mathrm{dist}(a_1, x)\cdot\mathrm{dist}(a_2, x)\cdot\ldots\cdot\mathrm{dist}(a_{r-1}, x)}{\mathrm{dist}(a_1, a_r)\cdot\mathrm{dist}(a_2, a_r)\cdot\ldots\cdot\mathrm{dist}(a_{r-1}, a_r)}\cdot\mathrm{dist}(y_0, b_r)$$

where $\mathrm{dist}(a_k, a_l) = \int_{a_k}^{a_l} s_X(x)\,dx_i$, y_0 is the first element of Y.

Another example is an adoption of a simple **rational interpolation**

$$\mathrm{dist}(\mathrm{y}_0,\mathrm{y}) = \frac{\sum_{k=1}^{r} w_k \cdot \mathrm{dist}(\mathrm{y}_0,\mathrm{b}_k)}{\sum_{k=1}^{r} w_k}, \quad w_k = \frac{1}{\left(\mathrm{dist}(\mathbf{x},\mathbf{a}_k)\right)^p}$$

where w_k is a weighting factor, inversely proportional to the vague distance of the observation and the k^{th} rule antecedent,

$$\mathrm{dist}(\mathbf{a}_k,\mathbf{x}) = \mathrm{dist}(\mathbf{x},\mathbf{a}_k) = \sqrt{\sum_{i=1}^{m}\left(\int_{\mathrm{a}_{k,i}}^{\mathrm{x}_i} s_{\mathrm{X}_i}(\mathrm{x}_i)\,d\mathrm{x}_i\right)^2},$$

$$\mathrm{dist}(\mathrm{y}_0,\mathrm{b}_k) = \int_{\mathrm{y}_0}^{\mathrm{b}_k} s_{\mathrm{Y}}(\mathrm{y})\,d\mathrm{y},$$

s_{X_i} is the i^{th} scaling function of the m dimensional antecedent universe,

s_{Y} is the scaling function of the one dimensional consequent universe,

$\mathbf{a}_k$ are the cores of the multidimensional fuzzy rule antecedents $\boldsymbol{A}_k$,

b_k are the cores of the one dimensional fuzzy rule consequents B_k,

$\mathbf{x}$ is the multidimensional crisp observation,

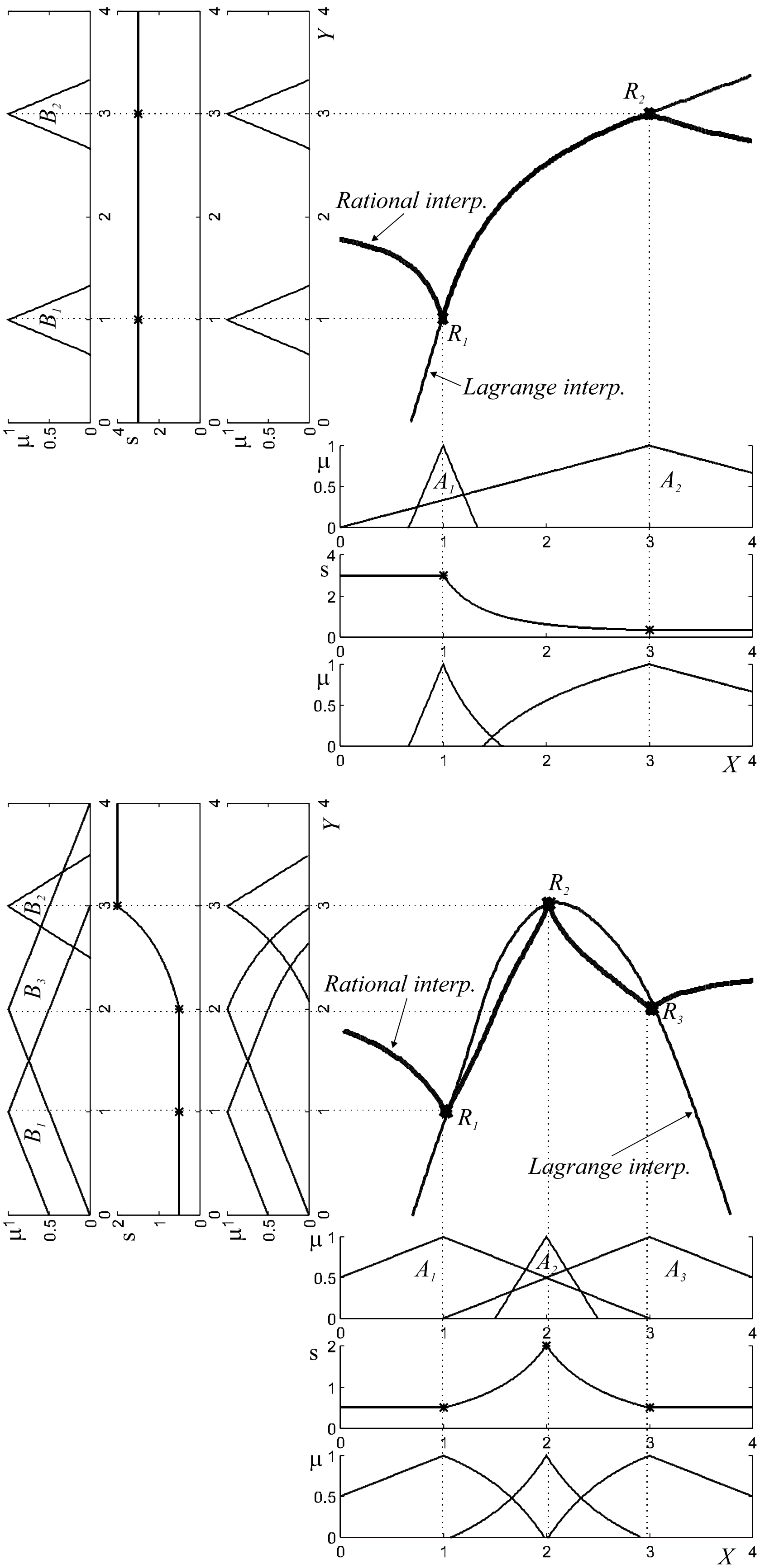
$R_{\mathrm{i}} = \boldsymbol{A}_{\mathrm{i}} \rightarrow \boldsymbol{B}_{\mathrm{i}}$ $\quad \mathrm{i}\in[1,2]$ are the fuzzy rules,

p is the sensitivity of the weighting factor for distant rules,

y_0 is the first element of the universe Y: $\mathrm{y}_0 \le \mathrm{y}\ \ \forall\ \mathrm{y}\in \mathrm{Y}$ (as Y is a one dimensional universe),

y is the one dimensional conclusion we are looking for.

Interpolation of three fuzzy rules (R_i:$A_i \rightarrow B_i$) in the approximated vague environment of the fuzzy rulebase, using the proposed rational interpolation (p=1) and the adopted Lagrange interpolation

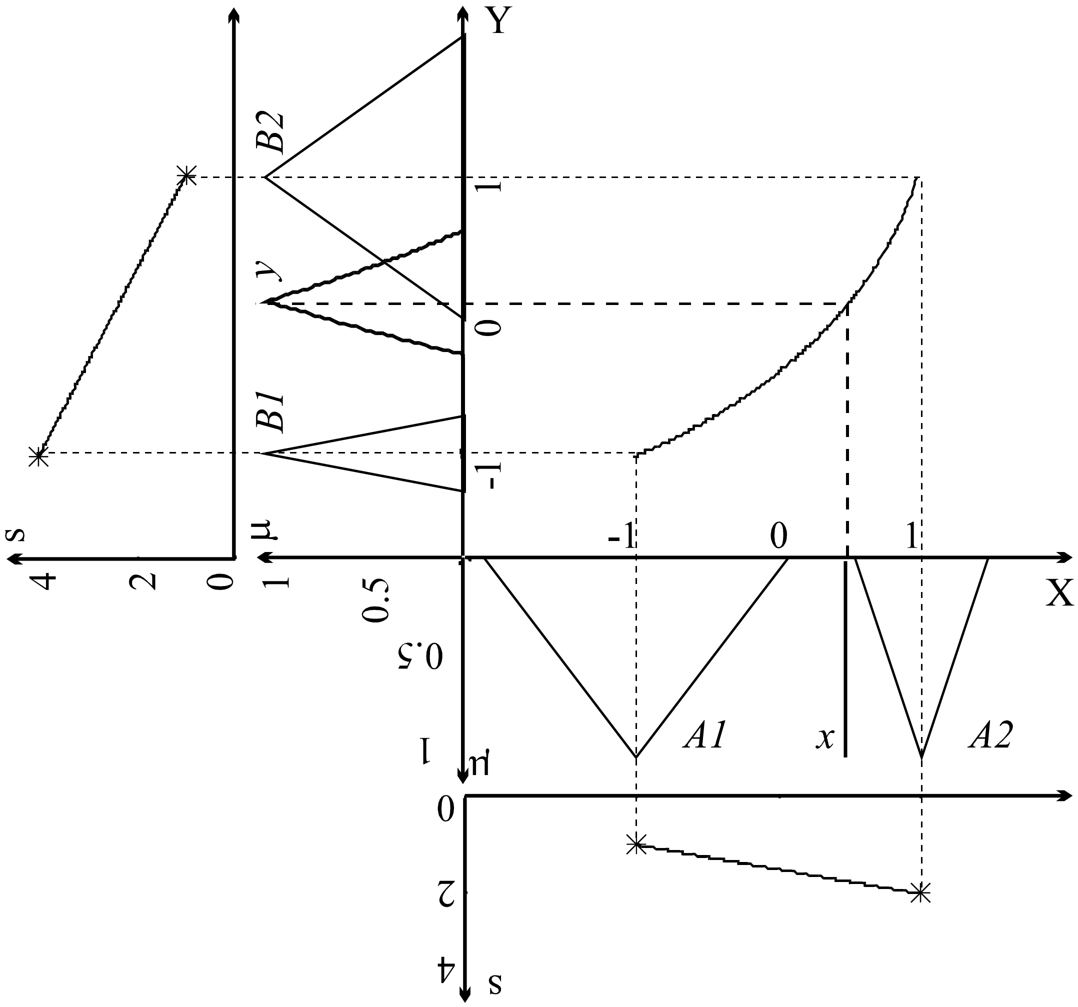


Generating the fuzzy conclusion

The vague conclusion calculated by rule interpolation is basically one point. Supposing that the terms in the fuzzy partition of the consequence universe describes all the main properties of the consequence universe and the scaling function approximated from this terms is proper, we can calculate the membership function of the **fuzzy conclusion as a level of similarity to the vague conclusion** in the vague environment of the consequence universe:

$$\mu_y(y) = 1 - \min\left\{\delta_{sy}(y, y_v), 1\right\} = 1 - \min\left\{\left|\int_{y}^{y_v} s_y(y)\,dy\right|, 1\right\},$$

where s_y is the scaling function of the consequence universe
y_v is the vague conclusion



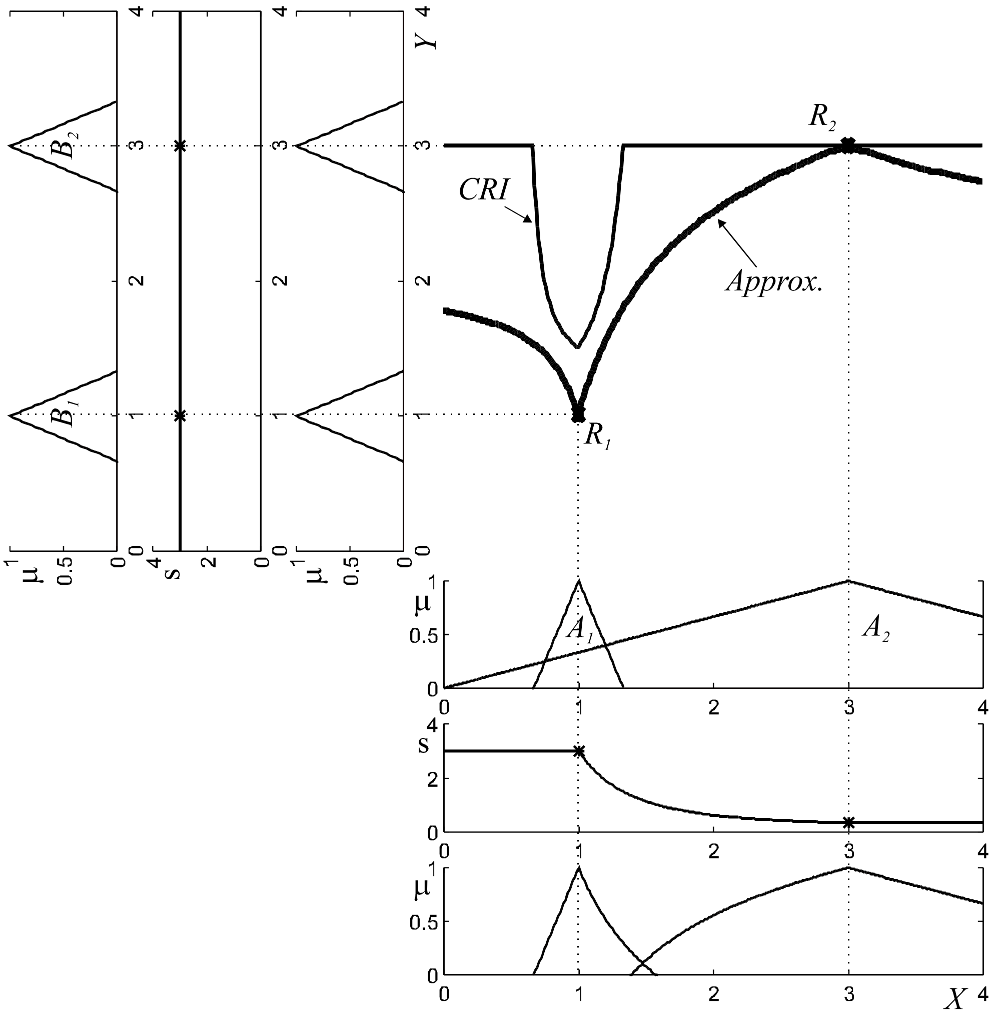
R_1: $A_1 \rightarrow B_1$, R_2: $A_2 \rightarrow B_2$

Comparing the crisp conclusion generated by approximate reasoning in the vague environment of the fuzzy rulebase to the crisp conclusion generated by the classical CRI

For comparing the crisp conclusions generated by the proposed approximate reasoning method to the classical **Compositional Rule of Inference** (CRI), we are choosing a representative one, the **min-max CRI** and the **centre of gravity defuzzification** method.

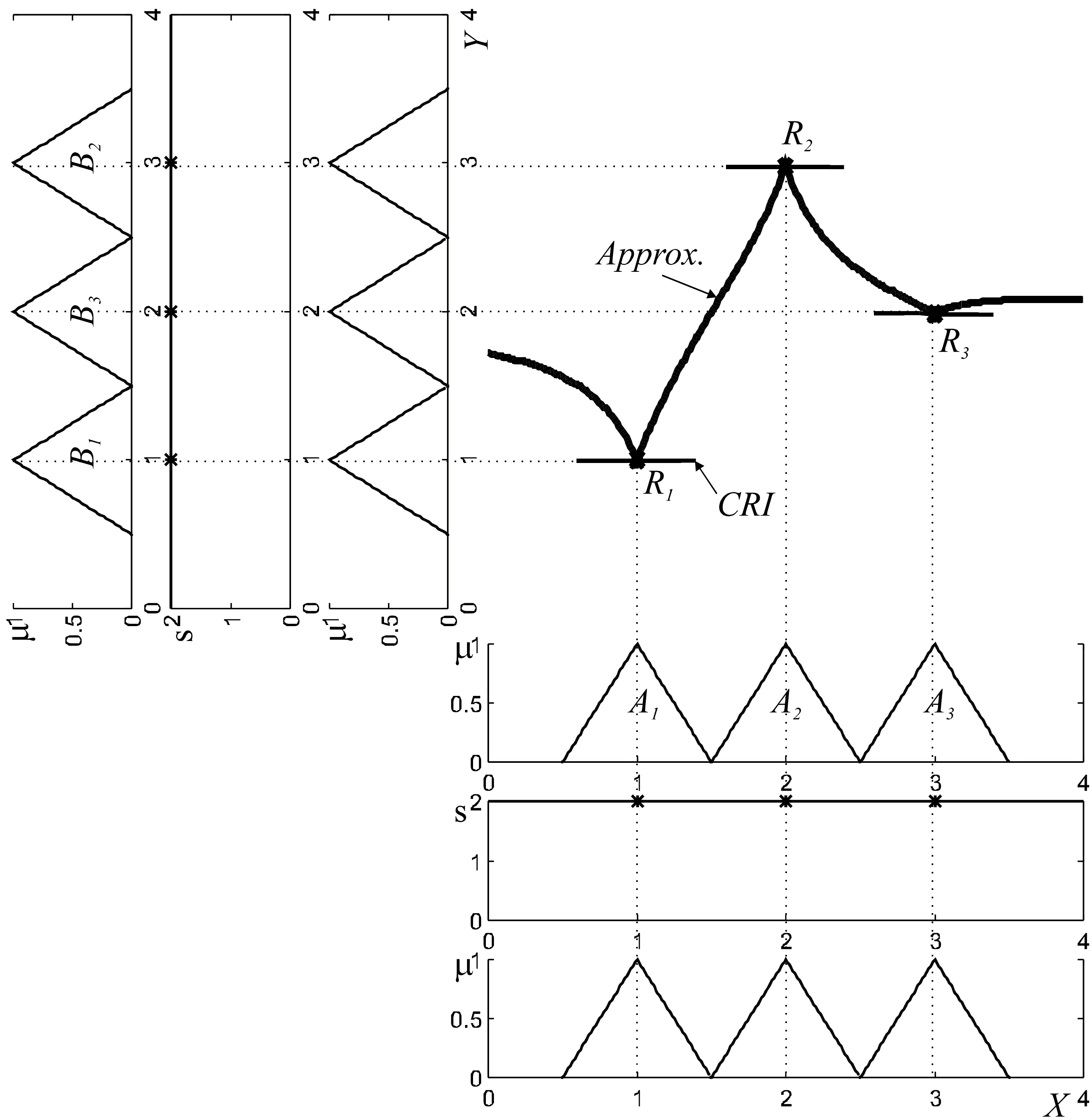
Results

The control function of the approximate fuzzy reasoning is always **fits the points of the fuzzy rules**. (A property of the interpolation function used) While the control function of the CRI is usually not fits these points. If an observation hits a rule antecedent exactly, than the conclusion generated by approximate fuzzy reasoning will be equal to the consequent part of the same fuzzy rule.



CRI
Approx.
R_1
R_2
A_1
A_2
B_1
B_2
X
Y
μ
s

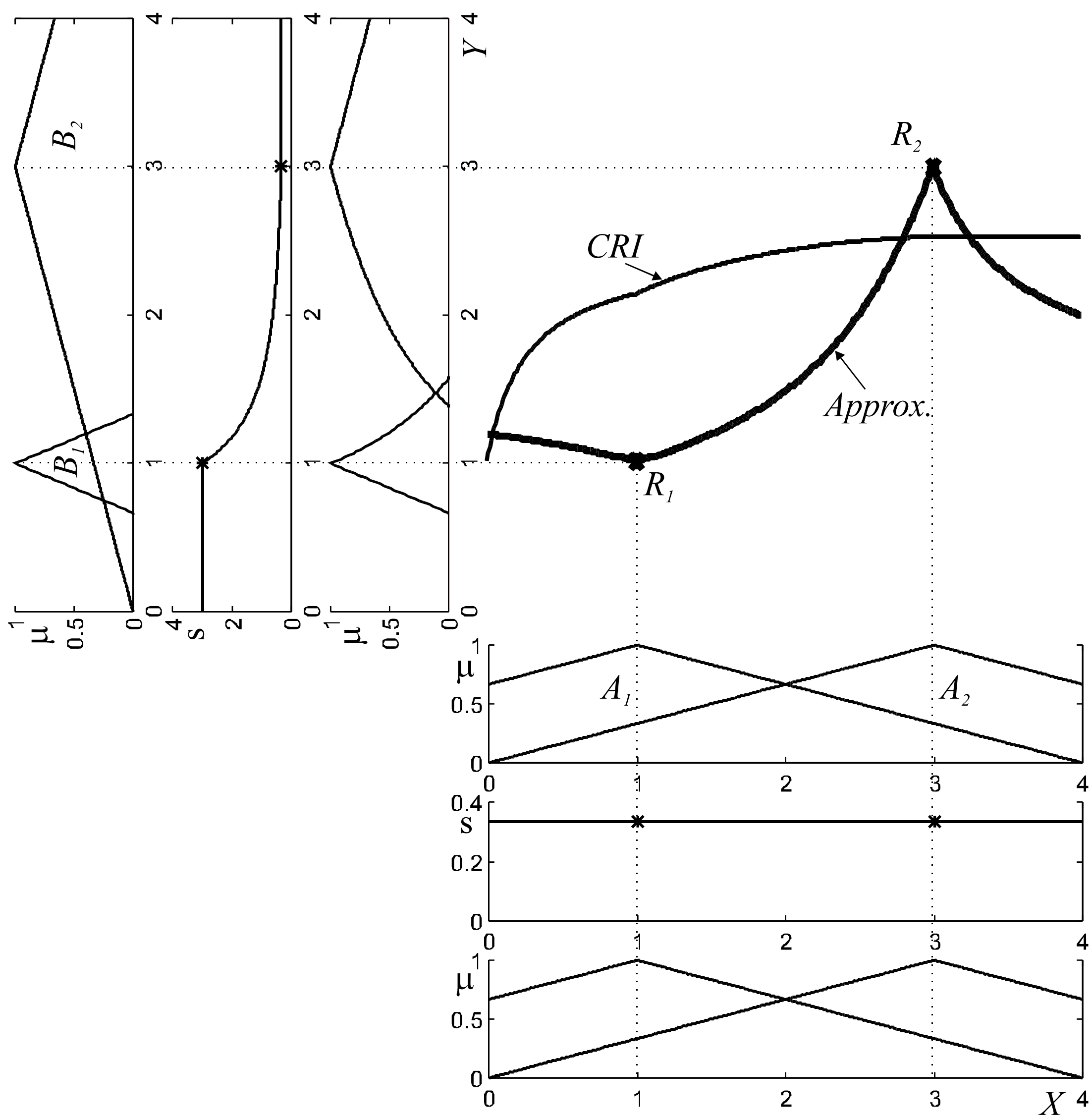
The approximate fuzzy reasoning method **gives conclusion for all the observations of the antecedent universe**, even if the fuzzy **rulebase is not complete**, while the CRI gives no conclusion if there are no overlapping between the observation and at least one of the rule antecedents (**approximate fuzzy reasoning with insufficient evidence**).



Interpolation of (R_i:$A_i \rightarrow B_i$), using rational interpolation (p=1) and the min-max CRI with the centre of gravity defuzzification

The **wide rule consequents has more influence** to the defuzzified crisp conclusion of the **CRI** (“wide consequents” are more “heavy” in the fuzzy conclusion), while using the method based on approximation in the vague environment of the fuzzy rulebase, the situation is the opposite. In other words it means, that using the **CRI**, in the crisp conclusion those **rules has dominance, whose consequent part is more “global”** (more “imprecise”, “fuzzy”, “wider”), in spite of the **approximate reasoning method**, where those **rules has the dominance, whose consequents are more “precise”** (more “crisp”, “narrower”).

It is a kind of philosophical question which rule needs more attendance, those, whose consequences are more global, or those, whose consequences are more precise.



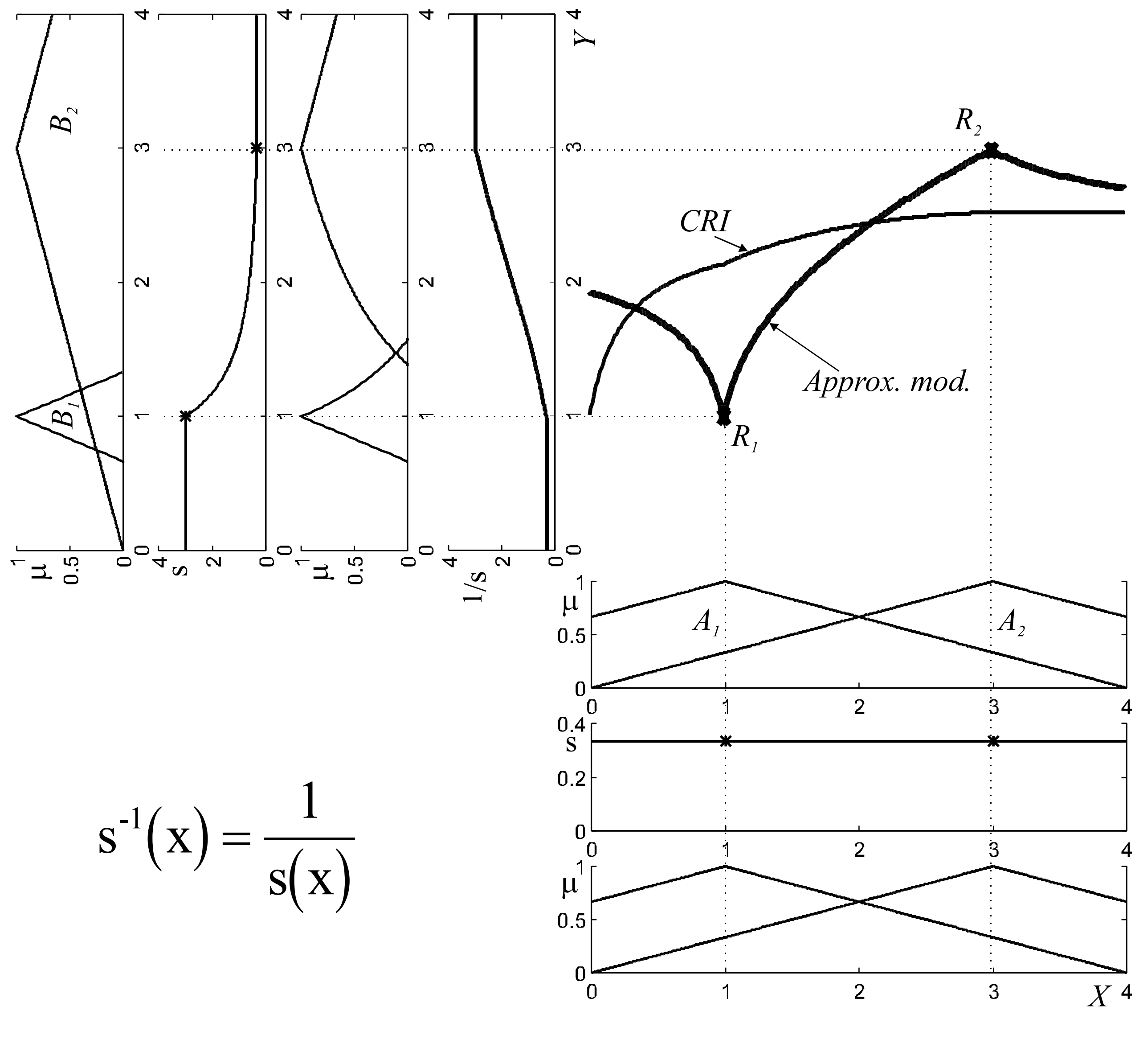
For changing this property, we suggest to introduce the concept of **inverse vague environment**. The main idea is **"inverting the density"** of a vague environment, by changing the original scaling function to its **inverse**. (dense regions ⇔ thin regions)

Inverting the **consequent vague environment** of the fuzzy rulebase, we can change the property of more attendance to the precise consequences, to more attendance to the global ones.

For **inverting the vague environment** (its scaling function s(x)) any **nondecreasing function** *i* can be used, which has the following property:

i: $[0,\infty] \rightarrow [0,\infty]$, $i(a) \leq i(b)$, $\forall$ a $\geq$ b, $\forall$ a,b $\in [0,\infty]$,

where $s^{-1}(x) = i\,[s(x)]$ $\forall$ x $\in X$, is the inverse scaling

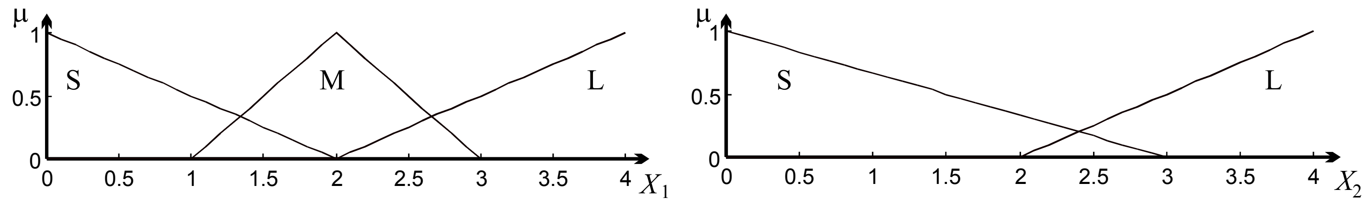


$$s^{-1}(x) = \frac{1}{s(x)}$$

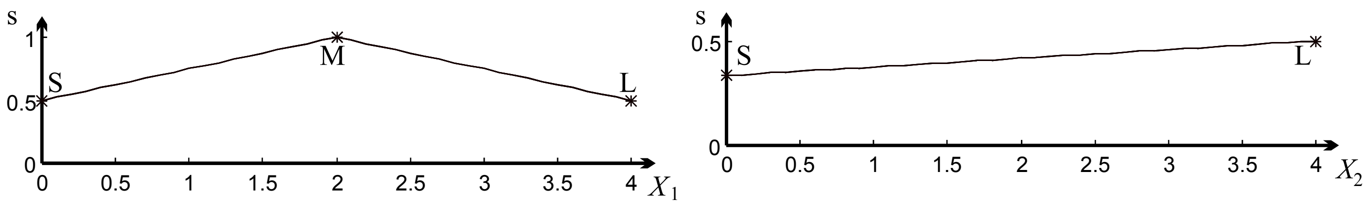
function.

X_1: X_2:

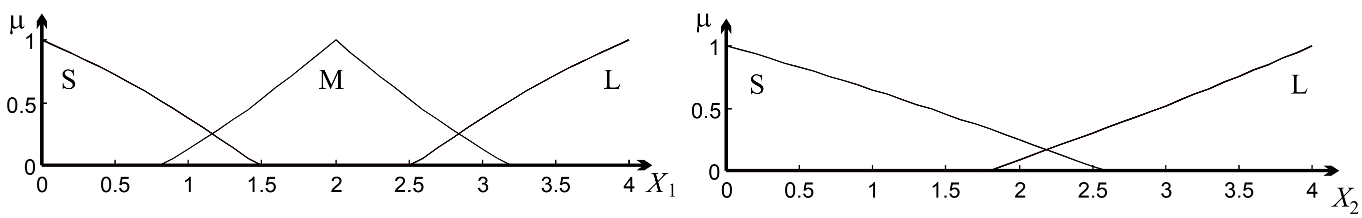
Original fuzzy partition:



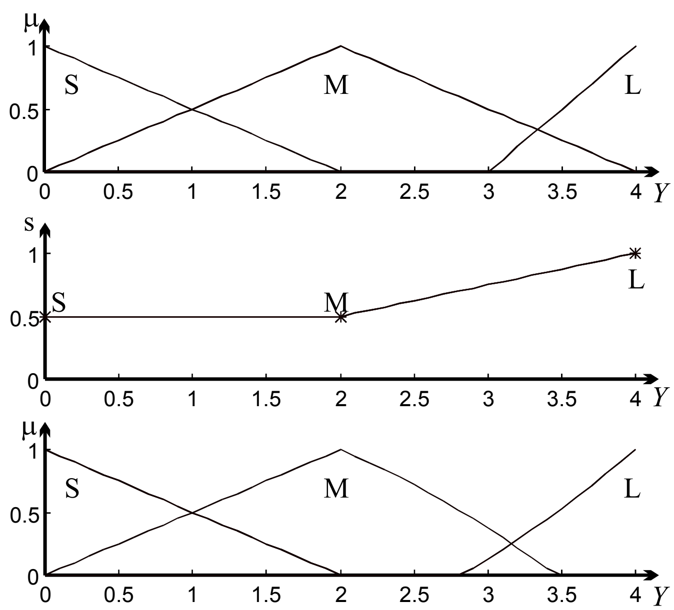
Approximate scaling function obtained from the original fuzzy partition:



Fuzzy partition described by the approximate scaling function:



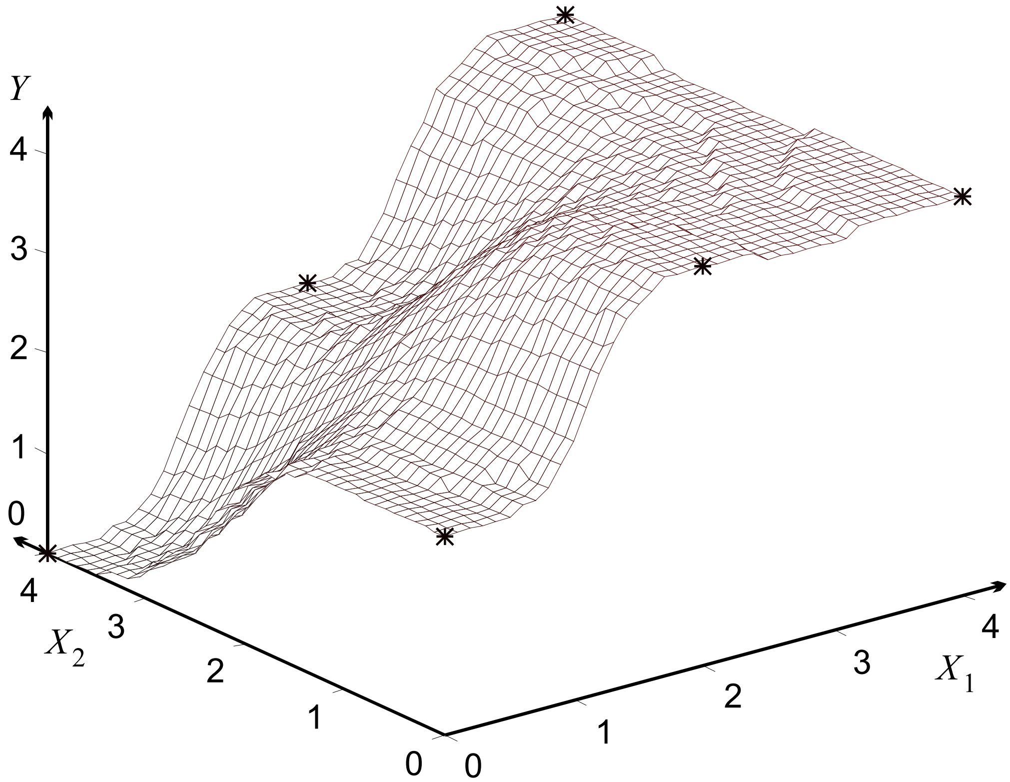
Y:



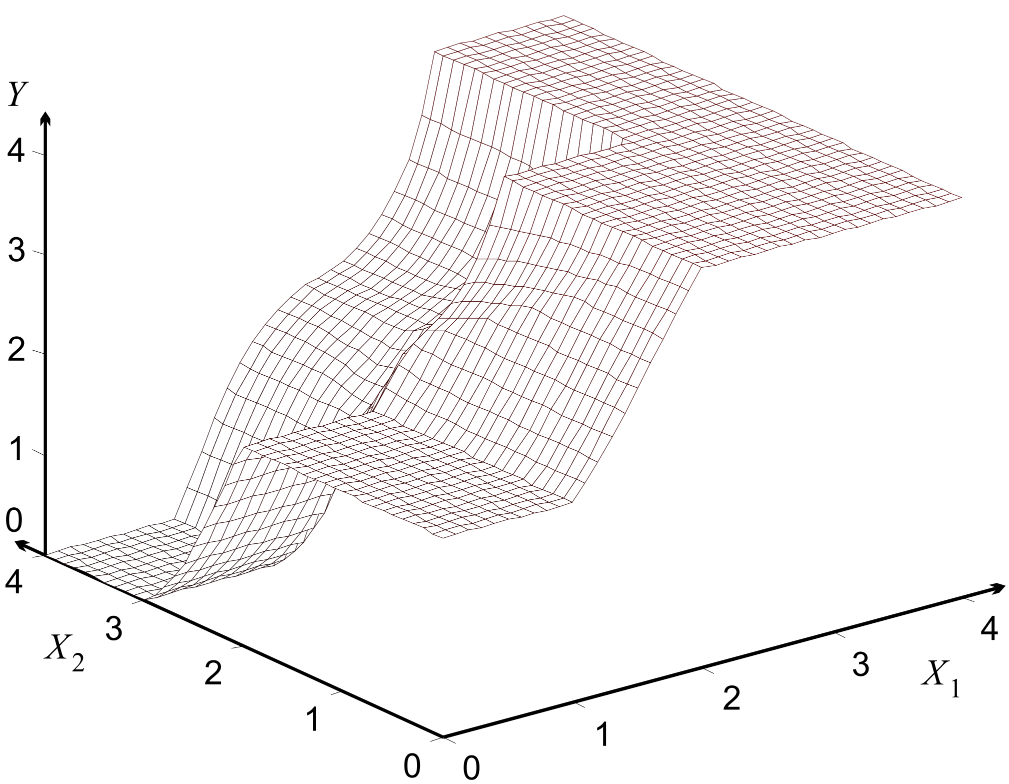
Rulebase:

if X_1=S **and** X_2=S **then** Y=M
if X_1=M **and** X_2=S **then** Y=L
if X_1=L **and** X_2=S **then** Y=L

if X_1=S **and** X_2=L **then** Y=S
if X_1=M **and** X_2=L **then** Y=M
if X_1=L **and** X_2=L **then** Y=L



Interpolation of six fuzzy rules in the approximated vague environment of the fuzzy rulebase (rational interpolation (p=1))



Max-min composition (CRI) with centre of gravity defuzzification

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