

Application of Interpolation-based Fuzzy Logic Reasoning in Behaviour-based Control Structures

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Abstract– Some difficulties emerging during the construction of fuzzy behaviour-based control structures are inherited from the type of the applied fuzzy reasoning. The fuzzy rule base requested for many classical reasoning methods needed to be complete. In case of fetching fuzzy rules directly from expert knowledge e.g. for the behaviour coordination module, the way of building a complete rule base is not always straightforward. One simple solution for overcoming the necessity of the complete rule base is the application of interpolation-based fuzzy reasoning methods, since interpolation-based fuzzy reasoning methods can serve usable (interpolated) conclusion even if none of the existing rules is hit by the observation. These methods can save the expert from dealing with derivable rules and help to concentrate on cardinal actions only. For demonstrating the applicability of the interpolation-based fuzzy reasoning methods in behaviour-based control structures a simple interpolation-based fuzzy reasoning method and its adaptation for behaviour-based control will be introduced briefly in this paper.

I. INTRODUCTION

In behaviour-based control systems (a good overview can be found in [3]), the actual behaviour of the system is formed as one of the existing behaviours (which fits best the actual situation), or as a kind of fusion of the known behaviours appeared to be the most appropriate to handle the actual situation. Beyond the construction of the behaviours, this structure has two other important tasks. The first is the decision, which behaviour is needed, or in case of behaviour fusion the determination of the necessity levels for each behaviour in solving the actual situation. The second is the way of the behaviour fusion. The first task, the behaviour coordination can be viewed as an actual system state approximation, where the actual system state is the set of the necessities of the known behaviours needed for handling the actual situation. The second is the fusion of the known behaviours based on their necessities.

In case of fuzzy behaviour based control structures both tasks are solved by fuzzy logic controllers. If the behaviours are also implemented on direct fuzzy logic controllers, the behaviours together with the behaviour fusion modules form a hierarchical fuzzy logic controller.

Since the classical fuzzy reasoning methods (e.g. compositional rule of inference) are demanding complete rule bases, all these rule bases have to build taking care to fill all the possible rules. In case if there is some rules are missing,

there are observations may exist which hit no rule in the rule base and therefore no conclusion is obtained. Having no conclusion at any level of the fuzzy behaviour based control structure is hard to explain. E.g. one solution could be to keep the last real conclusion instead of the missing one, but applying historical data automatically to fill undeliberately missing rules could cause unpredictable side effects.

Another solution for the same problem is the application of the interpolation-based fuzzy reasoning methods, where the derivable rules are deliberately missing. Since the rule base of a fuzzy interpolation-based controller, is not necessarily complete, it could contain the most significant fuzzy rules only without risking the chance of having no conclusion for some of the observations. In other words, during the construction of the fuzzy rule base, it is enough to concentrate on the cardinal actions; the “filling” rules (rules could be deduced from the others) can be omitted.

In the followings, first an approximate fuzzy reasoning method based on rational interpolation in the vague environment of the fuzzy rule base [4], [5], [6] will be introduced. The main benefit of the proposed method is its simplicity, as it could be implemented to be simple and quick enough to be applied in practical direct fuzzy logic control too. Then its adaptation to behaviour-based control structures together with a simple example will be discussed briefly.

II. INTERPOLATION-BASED FUZZY REASONING

One way of interpolative fuzzy reasoning is based on the concept of vague environment [2]. Applying the idea of the vague environment the linguistic terms of the fuzzy partitions can be described by scaling functions [2] and the fuzzy reasoning itself can be simply replaced by classical interpolation.

The concept of vague environment is based on the similarity or indistinguishability of the elements. Two values in the vague environment are ε -distinguishable if their distance is grater then ε . The distances in vague environment are weighted distances. The weighting factor or function is called *scaling function* (factor) [2]. Two values in the vague environment X are ε -distinguishable if

$$\varepsilon > \delta_s(x_1, x_2) = \left| \int_{x_2}^{x_1} s(x) dx \right|, \quad (1)$$

where $\delta_s(x_1, x_2)$ is the vague distance of the values x_1, x_2 and $s(x)$ is the scaling function on X .

For finding connections between fuzzy sets and a vague environment we can introduce the membership function $\mu_A(x)$ as a level of similarity \mathbf{a} to x , as the degree to which x is indistinguishable to \mathbf{a} [2]. The α -cuts of the fuzzy set $\mu_A(x)$ is the set which contains the elements that are $(1-\alpha)$ -indistinguishable from \mathbf{a} (see fig.1.): $\delta_s(\mathbf{a}, \mathbf{b}) \leq 1-\alpha$,

$$\mu_A(x) = 1 - \min\{\delta_s(\mathbf{a}, \mathbf{b}), 1\} = 1 - \min\left\{\int_a^b s(x) dx, 1\right\}. \quad (2)$$

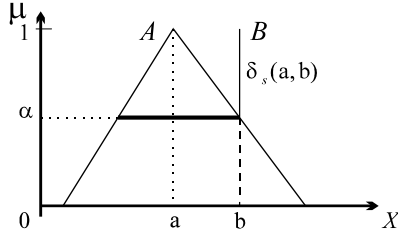


Fig.1. The α -cuts of $\mu_A(x)$ contains the elements that are $(1-\alpha)$ -indistinguishable from \mathbf{a}

It is very easy to realise (see fig.1.), that this case the vague distance of points \mathbf{a} and \mathbf{b} ($\delta_s(\mathbf{a}, \mathbf{b})$) is basically the *Disconsistency Measure* (S_D) of the fuzzy sets A and B (where B is a singleton):

$$S_D = 1 - \sup_{x \in X} \mu_{A \cap B}(x) = \delta_s(\mathbf{a}, \mathbf{b}) \text{ if } \delta_s(\mathbf{a}, \mathbf{b}) \in [0, 1], \quad (3)$$

where $A \cap B$ is the min t-norm, $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$ $\forall x \in X$.

It means, that we can calculate the disconsistency measures between member fuzzy sets of a fuzzy partition and a singleton, as vague distances of points in the vague environment of the fuzzy partition. The main difference between the disconsistency measure and the vague distance is, that the vague distance is a crisp value in range of $[0, \infty]$, while the disconsistency measure is limited to $[0, 1]$. That is why they are useful in interpolate reasoning with insufficient evidence.

Therefore if it is possible to describe all the fuzzy partitions of the primary fuzzy sets (the antecedent and consequent universes) of our fuzzy rule base by vague environments, and the observation is a singleton, we can calculate the “extended” disconsistency measures of the antecedent primary fuzzy sets of the rule base and the observation, and the “extended” disconsistency measures of the consequent primary fuzzy sets and the consequence (we are looking for) as vague distances of points in the antecedent and consequent vague environments.

The vague environment is described by its scaling function. For generating a vague environment of a fuzzy partition we have to find an appropriate scaling function, which describes the shapes of all the terms in the fuzzy partition. A fuzzy partition can be characterised by a single vague environment if and only if the membership functions of the terms fulfils the following requirement [2]:

$$s(x) = |\mu'(x)| = \left| \frac{d\mu}{dx} \right| \text{ exists iff} \quad (4)$$

$$\min\{\mu_i(x), \mu_j(x)\} > 0 \Rightarrow |\mu'_i(x)| = |\mu'_j(x)| \quad \forall i, j \in I$$

where $s(x)$ is the vague environment we are looking for.

Generally the above condition is not fulfilling, so the question is how to describe all fuzzy sets of the fuzzy partition with one “universal” scaling function. For this reason we propose to apply the concept of *approximate scaling function* [4], [5], [6].

The *approximate scaling function* is an approximation of the scaling functions describes the terms of the fuzzy partition separately [4], [5], [6].

If the vague environment of a fuzzy partition (the scaling function or the approximate scaling function) exists, the member sets of the fuzzy partition can be characterised by points in the vague environment. (In our case the points are characterising the cores of the terms, while the shapes of the membership functions are described by the scaling function itself.) If all the vague environments of the antecedent and consequent universes of the fuzzy rule base are exist, all the primary fuzzy sets (linguistic terms) used in the fuzzy rule base can be characterised by points in their vague environment. So the fuzzy rules (build on the primary fuzzy sets) can be characterised by points in the vague environment of the fuzzy rule base too. This case the approximate fuzzy reasoning can be handled as a classical interpolation task. Applying the concept of vague environment (the distances of points are weighted distances), any interpolation, extrapolation or regression methods can be adapted very simply for approximate fuzzy reasoning [4], [5], [6].

We suggest adapting the *Kóczy-Hirota interpolation* [7]. This method generates the conclusion as a weighted sum of the vague consequent values, where the weighting factors are inversely proportional to the vague distances of the observation and the corresponding rule antecedents:

$$\text{dist}(y_0, y) = \frac{\sum_{k=1}^r w_k \cdot \text{dist}(y_0, b_k)}{\sum_{k=1}^r w_k}, \quad w_k = \frac{1}{(\text{dist}(\mathbf{x}, \mathbf{a}_k))^p}, \quad (5)$$

where w_k is a weighting factor inversely proportional to the vague distance of the observation and the k^{th} rule antecedent,

$$\text{dist}(\mathbf{a}_k, \mathbf{x}) = \text{dist}(\mathbf{x}, \mathbf{a}_k) = \sqrt{\sum_{i=1}^m \left(\int_{a_{k,i}}^{x_i} s_{X_i}(x_i) dx_i \right)^2}, \quad (6)$$

$$\text{dist}(y_0, b_k) = \int_{y_0}^{b_k} s_Y(y) dy,$$

where s_{X_i} is the i^{th} scaling function of the m dimensional antecedent universe, s_Y is the scaling function of the one dimensional consequent universe, \mathbf{x} is the multidimensional crisp observation, \mathbf{a}_k are the cores of the multidimensional fuzzy rule antecedents A_k , b_k are the cores of the one dimensional fuzzy rule consequents B_k , $R_i = A_i \rightarrow B_i$ are the fuzzy rules, p is the sensitivity of the weighting factor for distant rules, y_0 is the first element of the one dimensional universe ($Y: y_0 \leq y \quad \forall y \in Y$), y is the one dimensional conclusion we are looking for.

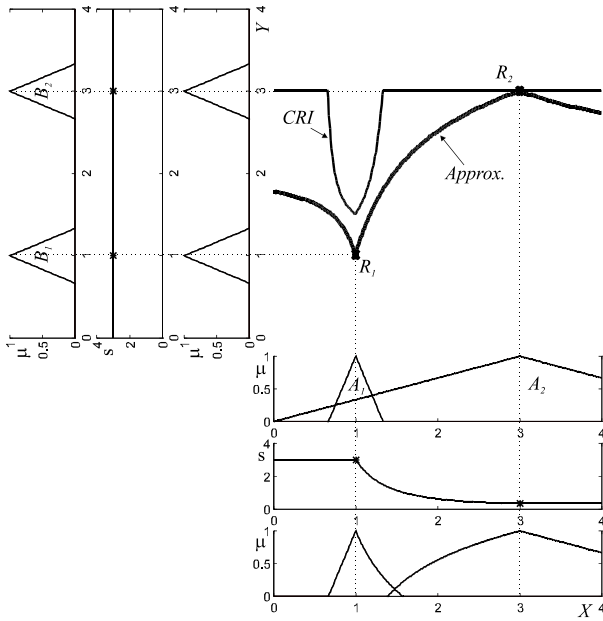


Fig.2. Interpolation of two fuzzy rules ($R_i: A_i \rightarrow B_i$) (see fig. 3. for notation).

A simple one-dimensional example for the approximate scaling function and the Kóczy-Hirota interpolation (6) is introduced on Fig. 2 and on Fig. 3. For comparing the crisp conclusions of the K-H interpolation and the classical methods, the conclusions generated by the max-min compositional rule of inference (CRI) and the centre of gravity defuzzification for the same rule base is also demonstrated on the figures. More detailed description of the proposed approximate fuzzy reasoning method can be found in [4], [5], [6].

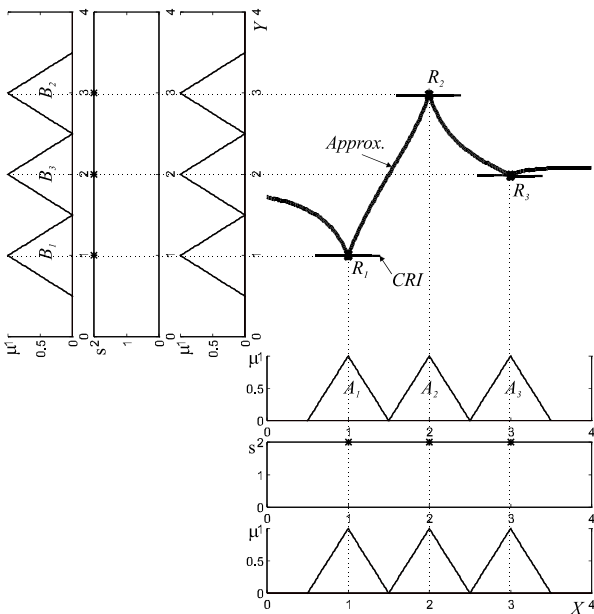


Fig.3. Interpolation of three fuzzy rules ($R_i: A_i \rightarrow B_i$) in the approximated vague environment of the fuzzy rule base, using the K-H interpolation ($p=1$) (Approx.) and the min-max. CRI with the centre of gravity defuzzification (CRI), where μ is the membership grade, and s is the scaling function.

III. THE APPLIED FUZZY BEHAVIOUR-BASED STRUCTURE

The main benefit of the interpolation-based fuzzy reasoning method, introduced in the previous chapter, is its simplicity. Applying look-up tables for pre-calculating the vague distances, it could be implemented to be simple and quick enough to fit the speed requirements of practical real-time direct fuzzy logic control systems, e.g. the requirements of fuzzy behaviour-based control too. The calculation efforts of many other interpolation-based fuzzy reasoning methods wasted for determining the exact membership shape of the interpolated fuzzy conclusion prohibits their practical application in real-time direct fuzzy logic control. The lack of the fuzziness in the conclusion is a disadvantage of the proposed method, but it has no influence in common applications where the next step after the fuzzy reasoning is the defuzzification.

In the followings a pure fuzzy behaviour-based control structure and the adaptation of the proposed interpolation-based fuzzy reasoning method will be discussed more detailed.

In case of pure fuzzy behaviour-based control structures all the main tasks of the behaviour-based control – the behaviour coordination, the behaviour fusion, and the behaviours themselves – are implemented on fuzzy logic controllers. (Such a structure is introduced on Fig.3.) Any of these controllers can apply the proposed interpolation-based approximate fuzzy reasoning method.

For demonstrating the main benefits of the interpolation-based fuzzy reasoning in behaviour-based control, in this paper we concentrate on the many cases most heuristic part of the structure, on the behaviour coordination.

The task of behaviour coordination is to determine the necessities of the known behaviours needed for handling the actual situation. In the suggested behaviour-based control structure, for this task the finite state fuzzy automaton is adapted (Fig.4.) [9]. This solution is based on the heuristic, that the necessities of the known behaviours for handling a given situation can be approximated by their suitability. And the suitability of a given behaviour in an actual situation can be approximated by the similarity of the situation and the prerequisites of the behaviour. (Where the prerequisites of the behaviour is the description of the situations where the behaviour is valid (suitable itself)). This case instead of determining the necessities of the known behaviours, the similarities of the actual situation to the prerequisites of all the known behaviours can be approximated.

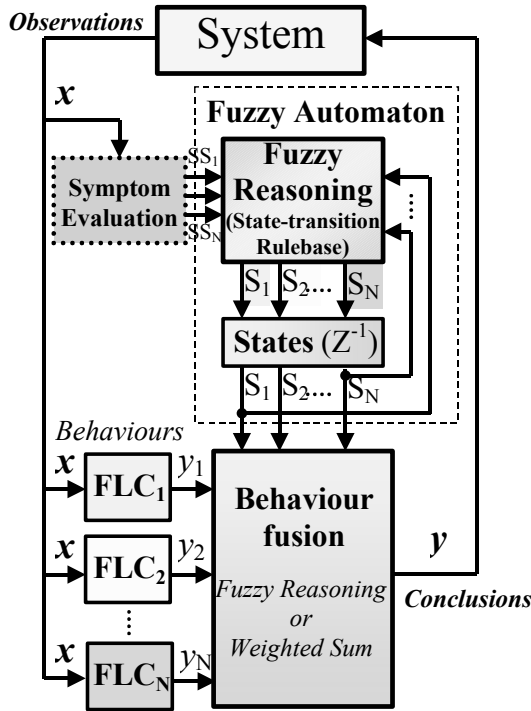


Fig.4. The suggested behaviour-based control structure.

Thus the first step of this kind of behaviour coordination is determining the similarities of the actual situation to the prerequisites of all the known behaviours – applying the terminology of fault classification; it is the symptom evaluation (see e.g. Fig.4.). The task of symptom evaluation is basically a series of similarity checking between an actual symptom (observations of the actual situation) and a series of known symptoms (the prerequisites – symptom patterns – of the known behaviours). These symptom patterns are characterising the systems states where the corresponding behaviours are valid. Based on these patterns, the evaluation of the actual symptom is done by calculating the similarity values of the actual symptom (representing the actual situation) to all the known symptoms patterns (the prerequisites of the known behaviours). There are many methods exist for fuzzy logic symptom evaluation. For example fuzzy classification methods e.g. the Fuzzy c-Means fuzzy clustering algorithm [1] can be adopted, where the known symptoms patterns are the cluster centres, and the similarities of the actual symptom to them can be fetched from the fuzzy partition matrix. On the other hand, having a simple situation, the fuzzy logic symptom evaluation could be a fuzzy rule based reasoning system itself.

One of the main difficulties of the system state approximation in behaviour coordination is the fact, that most cases the symptoms of the prerequisites of the known behaviours are strongly dependent on the actual behaviour of the system. Each behaviour has its own symptom structure. In other words for the proper system state approximation, the approximated system state is needed itself. A very simple way of solving this difficulty is the adaptation of fuzzy automaton. This case the state vector of the automaton is the approximated system state, and the state-transitions are driven by fuzzy reasoning (Fuzzy state-transition rule base on Fig.4.), as a decision based on the previous actual state (the

previous iteration step of the approximation) and the results of the symptom evaluation.

For demonstrating the simplicity of defining rule base for interpolation-based fuzzy reasoning, as an example, the state-transition rule base of the previously introduced fuzzy automaton style behaviour coordination module applied for user adaptive information retrieval system in [10] and [11] will be introduced briefly in the followings.

In our user adaptive information retrieval system example (introduced in [10] and [11] in more details) the user adaptivity is handled by combination of existing (off-line collected) human opinions (user models) in the function of the approximated similarity to the actual user opinions. As an analogy to the behaviour-based control applications, the different behaviours are the different existing user models, and the actual situation is the similarity of the actual user to the evaluators, originally gave the existing user models.

Based on our observations (inputs), the conclusion of the user feedback (the symptom evaluation about the state-transition to state i , SS_i for all the possible states $\forall i \in [1, N]$) and the previous state values S_i , we have to somehow estimate the new state values, the vector of the suitability of the existing user models. The heuristic we would like to implement in our example is very simple. If we already found a suitable model (S_i) and the user feedback is still supporting it (SS_i), we have to keep it even if the user feedback began to support some other models too. If there were no suitable model, but the user feedback began to support one, we have to pick it at once.

In case of interpolation-based fuzzy reasoning, the above heuristic can be simply implemented by the following state-transition rule base [10], [11]. For the i^{th} state variable S_i , $i \in [1, N]$ of the state vector S :

$$\text{If } S_i = \text{One} \quad \text{And } SS_i = \text{One} \quad \text{Then } S_i = \text{One} \quad (7.1)$$

$$\text{If } S_i = \text{Zero} \quad \text{And } SS_i = \text{Zero} \quad \text{Then } S_i = \text{Zero} \quad (7.2)$$

$$\text{If } S_i = \text{One} \quad \text{And } SS_i = \text{Zero} \quad \text{And } SS_k = \text{Zero} \quad \text{Then } S_i = \text{One} \quad (7.3) \\ \forall k \in [1, N], k \neq i$$

$$\text{If } S_i = \text{Zero} \quad \text{And } SS_i = \text{One} \quad \text{And } S_k = \text{Zero} \quad \text{And } SS_k = \text{Zero} \quad \text{Then } S_i = \text{One} \quad (7.4) \\ \forall k \in [1, N], k \neq i$$

$$\text{If } S_i = \text{Zero} \quad \text{And } SS_i = \text{One} \quad \text{And } S_k = \text{One} \quad \text{And } SS_k = \text{One} \quad \text{Then } S_i = \text{Zero} \quad (7.5) \\ \exists k \in [1, N], k \neq i$$

where SS_i is the conclusion of the symptom evaluation about the state-transition to state i , $\forall i \in [1, N]$, N is the number of known behaviours (state variables). The structure of the state-transition rules is similar for all the state variables. Zero and One are linguistic labels of fuzzy sets (linguistic terms) representing high and low similarity. The interpretations of the Zero and One fuzzy sets can be different in each S_i , SS_i universes.

Please note that rule base (7) is sparse. It contains the main rules for the following straightforward goals only: Rule (7.1) simply keeps the previously chosen state values in the case if the symptom evaluation also agrees. The rule (7.2) has the opposite meaning, if the state values were not chosen, and moreover the symptom evaluation is also disagrees the state value should be suppressed. The rule (7.3) keeps the already

selected state values (previous approximation), even if the symptom evaluation disagrees, if it has no better “idea”. Rules (7.4) and (7.5) have the task of ensuring the relatively quick convergence of the system to the sometimes unstable (changeable) situations, as new state variables which seem to be fit, can be chosen in one step, if there is no previously chosen state, which is still accepted by the symptom evaluation (7.4). (Rule (7.5) has the task to suppress this selection in the case if exists a still acceptable state which has already chosen.) The goal of this heuristic is to gain a relatively quick convergence for the system to fit the opinions of the actual user, if there is no state value high enough to be previously accepted. This quick convergence could be very important in many application areas e.g. in case of an on-line user adaptive selection system introduced in [10], where the user feed-back information needed for the state changes are very limited.

Some state changes of the fuzzy automaton in the function of the conclusion of the symptom evaluation (SS_1, SS_2) for the two states case (applying the state-transition rule base (7)) are visualised on Fig.5. and Fig.6.

If we count the rules of the classical fuzzy reasoning for the same strategy we find, that the complete rule base needs 16 rules for the two state case (as we have four observation universes (S_1, SS_1, S_2, SS_2) each with two terms fuzzy partitions (Zero, One)), while the sparse rule base (7) contains 7 rules only in the same two state case.

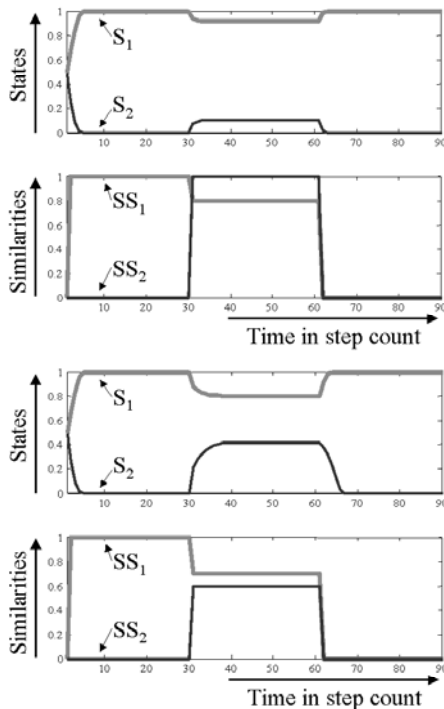


Fig.5. Do not “pick up” a new state if the previous approximation is still adequate.

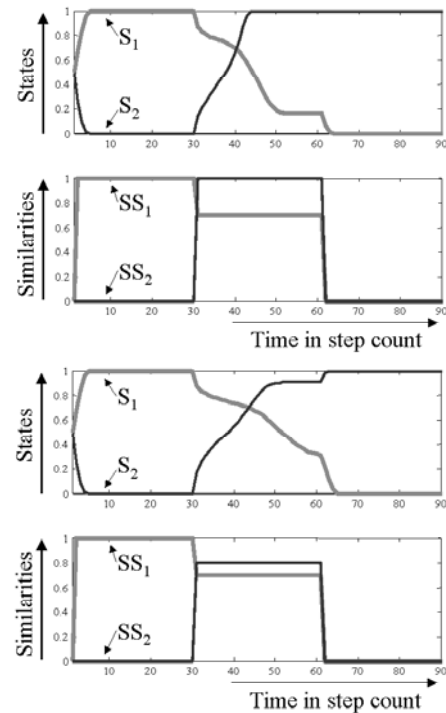


Fig.6. But “pick it up” if it seems better, or at least as good as the previous was.

IV. CONCLUSIONS

The goal of this paper was to introduce an interpolation-based fuzzy reasoning method, which could be implemented to be simple and quick enough to fit the requirements of behaviour-based control structures in real-time direct fuzzy logic control systems.

The suggested approximate fuzzy reasoning method based on K-H interpolation in the vague environment of the fuzzy rule base gives an efficient way for designing direct fuzzy logic control applications.

The lack of the fuzziness in the conclusion is a disadvantage of the proposed method, but it has no influence in common applications where the next step after the fuzzy reasoning is the defuzzification.

For demonstrating the efficiency of the interpolation-based fuzzy reasoning in behaviour-based control, a fuzzy behaviour-based control structure based on fusion of different known behaviours in the function of their actual necessities approximated by fuzzy automaton is also introduced in this paper briefly.

To give some guidelines for interpolation-based fuzzy reasoning rule base design, some highlights of the behaviour coordination rule base of a user adaptive information retrieval system application is also introduced in this paper.

The implementation of interpolation-based fuzzy reasoning methods in behaviour-based control structures simplifies the task of fuzzy rule base creation. Since the rule base of a fuzzy interpolation-based controller, is not necessarily complete, it could contain the most significant fuzzy rules only without risking the chance of having no conclusion for some of the observations. In other words, during the construction of the fuzzy rule base, it is enough to concentrate on the cardinal actions; the “filling” rules (rules could be deduced from the

others) could be deliberately omitted. Thus, compared to the classical fuzzy compositional rule of inference, the number of the fuzzy rules needed to be handled during the design process could be dramatically reduced.

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