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Extending the Fuzzy Rule Interpolation "FIVE" by Fuzzy observation

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Some difficulties emerging during the construction of fuzzy rule bases are inherited from the type of the applied fuzzy reasoning. In fuzzy systems, when classical methods (e.g. the Compositional Rule of Inference) are applied, the completeness of the fuzzy rule base is required to generate meaningful output. This means, that the fuzzy rule base has to cover all possible inputs. The way of building a complete rule base is not always straightforward. One simple solution to handle sparse fuzzy rule bases and to make infer reasonable output is the application of fuzzy rule interpolation (FRI) methods. On the other hand most of the FRI methods share the burden of high computational demand. However there is a method "FIVE" (Fuzzy Interpolation based on Vague Environment, originally introduced in [8], [11] and [12]) which is simple and quick enough to fit even the requirements of direct control, where the conclusions are applied as real-time control actions, too. Beyond the simplicity and therefore the high reasoning speed, "FIVE" has two obvious drawbacks, the lack of the fuzziness on the observation and conclusion side. The main contribution of this paper is the introduction of a way for handling fuzzy observations by extending the original "FIVE" concept with the ability of merging vague environments.

1 Introduction

Since the classical fuzzy reasoning methods (e.g. compositional rule of inference) are demanding complete rule bases, the classical rule base construction claims a special care of filling all the possible rules. In case if there are some rules missing, observations may exist which hit no rule in the rule base and therefore no conclusion is obtained. Having no conclusion in a fuzzy control structure is hard to explain. E.g. one solution could be to keep the last real conclusion instead of the missing one, but applying historical data automatically to fill undeliberately missing rules could cause unpredictable side effects. Another solution for the same problem is the application of the fuzzy rule interpolation (FRI) methods, where the derivable rules are deliberately missing. Since the rule base of an FRI controller is not necessarily complete, it could contain the most significant fuzzy rules only without risking the chance of having no conclusion for some of the observations. On the other hand most of the FRI methods are sharing the burden of high computational demand, e.g. the task of searching for the two closest surrounding rules to the observation, and calculating the conclusion at least in some characteristic α-cuts. Moreover in some methods the interpretability of the fuzzy conclusion gained is also not straightforward [7]. There have been a lot of efforts to rectify the interpretability of the interpolated fuzzy conclusion [17]. In [1] Baranyi *et al.* give a comprehensive overview of the recent existing FRI methods (namely the α-cut, modified α-cut and generalised fuzzy interpolation methods). Beyond these problems, some of the FRI methods are originally defined for one dimensional input space, and need special extension for the multidimensional case (e.g. [3]-[4]). In [20] Wong *et al.* gave a comparative overview of the recent multidimensional input space capable FRI methods. In [3] Jenei introduced a way for axiomatic treatment of the FRI methods. In [14] Perfilieva studies the solvability of fuzzy relation equations as the solvability of interpolating and approximating fuzzy functions with respect to a given set of fuzzy rules (e.g. fuzzy data as ordered pairs of fuzzy sets).

The high computational demand, mainly the search for the two closest surrounding rules to an arbitrary observation in the multidimensional antecedent space makes many of these methods hardly suitable for real-time applications. Some FRI, e.g. the method introduced by Jenei *et al.* in [4], eliminate the search for the two closest surrounding rules by taking all the rules into consideration, and therefore speed up the reasoning process. On the other hand, keeping the goal of constructing fuzzy conclusion, and not simply speeding up the reasoning process, they still require some additional (or repeated) computational steps for the elements of the level set (or at least some relevant α levels) to get the fuzzy conclusion.

A rather different application oriented aspect of the FRI emerges in the concept of “FIVE”. The fuzzy reasoning method “FIVE” (Fuzzy Interpolation based on Vague Environment, originally introduced in [8], [11] and [12]) was developed to fit the speed requirements of direct fuzzy control, where the conclusions of the fuzzy controller are applied directly as control actions in a real-time system (see e.g. a downloadable and runable code of a real-time vehicle path tracking and collision avoidance control at [21]).

Beyond the simplicity and therefore the high reasoning speed, the FIVE has two obvious drawbacks, the lack of the fuzziness on the observation and conclusion side. The reason is this deficiency is inherited from the nature of the applied vague environment, which describes the indistinguishability of two points and therefore the similarity of a fuzzy set and a singleton only. The lack of the fuzziness on the conclusion side has a small influence on common applications where the next step after the fuzzy reasoning is the defuzzification. On the other hand, the lack of the fuzziness on the observation side can restrict applicability of the method.

In the followings, a way of merging vague environments and therefore the extension of the original FIVE concept with the ability of handling fuzzy observations will be introduced.

2 The concept of vague environment

The FIVE FRI method is based on the concept of the vague environment [5]. Applying the idea of the vague environment the linguistic terms of the fuzzy partitions can be described by scaling functions [5] and the fuzzy reasoning itself can be replaced by classical interpolation. The concept of a vague environment is based on the similarity or indistinguishability of the considered elements. Two values in a vague environment are ε-distinguishable if their distance is greater than ε. The distances in a vague environment are weighted distances. The weighting factor or function is called *scaling function (factor)* [5].

Two values in the vague environment *X* are *ε-indistinguishable* if

$$\varepsilon \geq \delta_s(x_1, x_2) = \left| \int_{x_2}^{x_1} s(x)dx \right| , \tag{1}$$

where $\delta_s(x_1, x_2)$ is the scaled distance of the values x_1, x_2 and $s(x)$ is the scaling function on *X*.

For finding connections between fuzzy sets and a vague environment the membership function $\mu_A(x)$ can be introduced as indicating level of similarity of *x* to a specific element *a* that is a representative or prototypical element of the fuzzy set $\mu_A(x)$, or, equivalently, as the degree to which *x* is indistinguishable from *a* (2) [5]. The α-cuts of the fuzzy set $\mu_A(x)$ are the sets which contain the elements that are (1−α)-indistinguishable from *a* (see Fig.1. also):

$$1-\alpha \geq \delta_s(\mathrm{a},\mathrm{b}) \ , \quad \mu_A(x) = 1-\min\{\delta_s(\mathrm{a},\mathrm{b}),1\} = 1-\min\left\{\left|\int_{\mathrm{a}}^{\mathrm{b}} s(x)dx\right|,1\right\} . \tag{2}$$

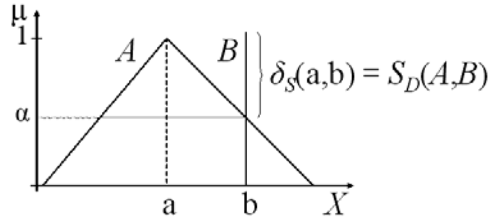

Fig.1. The α-cuts of $\mu_A(x)$ contain the elements that are (1−α)-indistinguishable from *a*.

In this case (see Fig.1), the scaled distance of points a and b ($\delta_s(\mathrm{a},\mathrm{b})$) is the *Disconsistency Measure* (S_D) (mentioned and studied among other distance measures in [19] by Turksen *et al.*) of the fuzzy sets *A* and *B* (where *B* is a singleton):

$$S_D(A,B) = 1 - \sup_{x\in X} \mu_{A\cap B}(x) = \delta_s(\mathrm{a},\mathrm{b}) \text{ if } \delta_s(\mathrm{a},\mathrm{b}) \in [0,1] \,, \tag{3}$$

where $A \cap B$ notes the min t-norm, $\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)]$ $\forall x \in X$.

Taking into account the most common way of building a traditional fuzzy logic controller, where the first step is defining the fuzzy partitions on the antecedent and consequent universes by setting up the linguistic terms and then based on these terms building up the fuzzy rule base, the concept of vague environment [5] is straightforward. The goal of the fuzzy partitions is to define indistinguishability, or vagueness on the different regions of the input, output universes. This situation is clearer, if intentionally Ruspini partitions are chosen and the cores of the linguistic terms are set only (see e.g. Fig.2). The designer has no intention to specify particular fuzzy sets, but the vagueness of the terms and therefore the vagueness of the rules build from them.

The vague environment is characterised by its scaling function. For generating a vague environment of a fuzzy partition an appropriate scaling function is needed, which describes the shapes of all the terms in the fuzzy partition. A fuzzy partition can be characterised by a single vague environment if and only if the membership functions of the terms fulfil the following requirement [5]:

$$\mathrm{s}(x) = |\mu'(x)| = \left|\frac{d\mu}{dx}\right| \text{ exists iff } \min\{\mu_i(x), \mu_j(x)\} > 0 \Rightarrow |\mu'_i(x)| = |\mu'_j(x)| \,, \tag{4}$$

$\forall i, j \in I$, where s(x) is the scaling function of the vague environment (see e.g. on fig.2.).

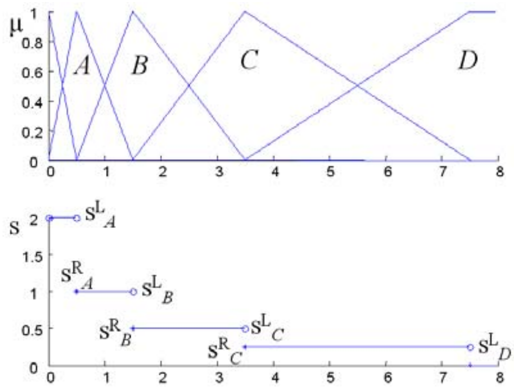

Fig.2. A Ruspini fuzzy partition and its scaling function.

3 Approximate scaling function

Generally condition (4) is not fulfilled, so the question is how to describe all fuzzy sets of the fuzzy partition with one "universal" scaling function. For this task the concept of an *approximate scaling function*, as an approximation of the scaling functions describing the terms of the fuzzy partition separately is proposed in [8], [11], [12].

The concept of an approximate scaling function is based on the assumption that the original goal of setting up a fuzzy partition was to characterise a scaling on a universe by some given points (member sets of the fuzzy partition), where the scaling factor of the universe is known. This case, as a general way of describing scaling on a universe, the member sets of the fuzzy partition can be restricted to triangular (trapezoidal) shaped terms. Supposing that the fuzzy terms are triangles, each fuzzy term can be characterised by three values (by a triple), by the values of the left and the right scaling factors and the value of its core point (see e.g. on Fig.3). Having these cardinal points, as an approximate scaling function, the scaling function can be simply interpolated. In [8], [11], [12] the following non-linear formula was suggested for interpolation of the corresponding scaling factors between the neighbouring terms (see e.g. on Fig.4):

$$s(x)=\begin{cases}\dfrac{w_i}{(d_i+1)^{k\cdot w_i}-1}\cdot\left(\dfrac{(d_i+1)^{k\cdot w_i}}{(x-x_i+1)^{k\cdot w_i}}-1\right)+s^L_{i+1} & \left|\, s^R_i \ge s^L_{i+1},\right. \\ \dfrac{w_i}{(d_i+1)^{k\cdot w_i}-1}\cdot\left(\dfrac{(d_i+1)^{k\cdot w_i}}{(x_{i+1}-x+1)^{k\cdot w_i}}-1\right)+s^R_i & \left|\, s^R_i < s^L_{i+1},\right. \end{cases} \quad x\in[x_i,x_{i+1}),\ \forall\, i\in[1,n-1] \qquad \textbf{(5)}$$

where $s(x)$ is the approximate scaling function; x_i is the core of the i[th] term of the approximated fuzzy partition; s_i^L, s_i^R are the left and right side scaling factors of the i[th] triangle shaped term, n is the number of the terms in the approximated fuzzy partition, $w_i = \left|s^L_{i+1} - s^R_i\right|$, $d_i = x_{i+1} - x_i$, $\forall\, i \in [1, n-1]$; and k, k>0 is the sensitivity factor for neighbouring scaling factor differences.

For a detailed discussion of questions related to approximate scaling functions see [8], [11], [12].

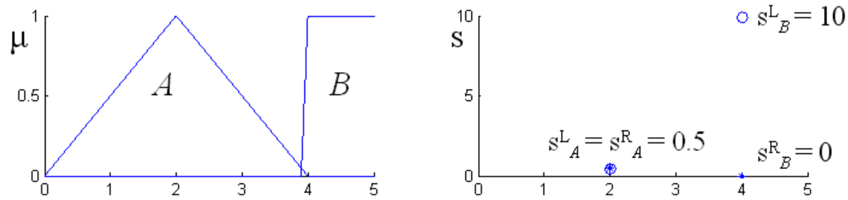

Fig.3. Fuzzy partitions consisting of triangular fuzzy sets can be characterised by triples, by the values of the left s^L and the right s^R scaling factors and the cores.

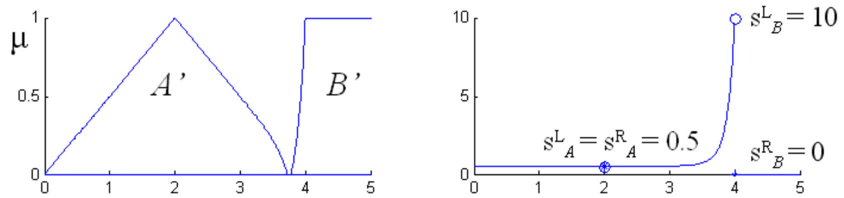

Fig.4. Approximate scaling function generated by non-linear interpolation (5) (k=1) of the fuzzy partition shown on Fig.3, and the partition as the approximate scaling function describes it (A',B').

4 Shepard interpolation for fuzzy reasoning: "FIVE"

The main idea of the FRI method "FIVE" (Fuzzy Interpolation based on Vague Environment) can be summarised in the followings:

a) If the vague environment of a fuzzy partition (the scaling function or at least the approximate scaling function) exists, the member sets of the fuzzy partition can be characterised by points in that vague environment. (These points are indicating the cores of the fuzzy terms, while the membership functions are described by the scaling function itself.)

b) If all the vague environments of the antecedent and consequent universes of the fuzzy rule base exist, all the primary fuzzy sets (linguistic terms) compounding the fuzzy rule base can be characterised by points in their vague environment. Therefore the fuzzy rules (built-up from the primary fuzzy sets) can be characterised by points in the vague environment of the fuzzy rule base too. In this case the approximate fuzzy reasoning can be handled as a classical interpolation task.

c) Applying the concept of vague environments (the distances of points are weighted distances), any crisp interpolation, extrapolation, or regression method can be adapted very simply for approximate fuzzy reasoning [8], [11] and [12].

Because of its simple multidimensional applicability, for interpolation-based fuzzy reasoning in this paper the adaptation of the *Shepard operator* based interpolation (first introduced in [15]) is suggested. Beside the existing deep application oriented investigation of the Shepard operator e.g. [2], it is also successfully applied in the *Kóczy-Hirota fuzzy interpolation* [6]. (The stability and the approximation rate of the Shepard operator based Kóczy-Hirota fuzzy interpolation is thoroughly studied in [18] and [16].) The Shepard interpolation method for arbitrarily placed bivariate data was introduced as follows [15]:

$$S_0(f,x,y)=\begin{cases} f_k & \text{if } (x,y)=(x_k,y_k) \text{ for some } k, \\ \left(\sum_{k=0}^{n} f(x_k,y_k)/d_k^{\lambda}\right) \Big/ \left(\sum_{k=0}^{n} 1/d_k^{\lambda}\right) & \text{otherwise,} \end{cases} \quad \textbf{(6)}$$

where measurement points x_k, y_k ($k\in[0,n]$) are irregularly spaced on the domain of $f\in\Re^2\to\Re$, $\lambda>0$, and $d_k=\left[(x-x_k)^2+(y-y_k)^2\right]^{1/2}$. This function can be typically used when a surface model is required to interpolate scattered spatial measurements.

The adaptation of the Shepard interpolation method for interpolation-based fuzzy reasoning in the vague environment of the fuzzy rule base is straightforward by substituting the Euclidian distances d_k by the scaled distances $\delta_{s,k}$:

$$\delta_{s,k}=\delta_s(\mathbf{a}_k,\mathbf{x})=\left[\sum_{i=1}^{m}\left(\int_{\mathrm{a}_{k,i}}^{\mathrm{x}_i} s_{\mathrm{X}_i}(\mathrm{x}_i)\,d\mathrm{x}_i\right)^2\right]^{1/2}, \quad \textbf{(7)}$$

where s_{X_i} is the i^{th} scaling function of the m dimensional antecedent universe, $\mathbf{x}$ is the m dimensional crisp observation and $\mathbf{a}_k$ are the cores of the m dimensional fuzzy rule antecedents $\boldsymbol{A}_k$.

Thus in case of singleton rule consequents the fuzzy rules R_k has the following form:

$$\textbf{If } x_1 = A_{k,1} \textbf{ And } x_2 = A_{k,2} \textbf{ And} \ldots \textbf{And } x_m = A_{k,m} \textbf{ Then } y = c_k \tag{8}$$

by substituting (7) to (6) the conclusion of the interpolative fuzzy reasoning can be obtained as:

$$y(\mathbf{x}) = \begin{cases} c_k & \text{if } \mathbf{x} = \mathbf{a}_k \text{ for some } k, \\ \left(\sum_{k=1}^{r} c_k / \delta_{s,k}^{\lambda}\right) \Big/ \left(\sum_{k=1}^{r} 1/\delta_{s,k}^{\lambda}\right) & \text{otherwise.} \end{cases} \tag{9}$$

The interpolative fuzzy reasoning (9) can be extend simply to be able to handle fuzzy conclusions by introducing the vague environment (scaling function) of the consequence universe. In this case the fuzzy rules R_k has the following form:

$$\textbf{If } x_1 = A_{k,1} \textbf{ And } x_2 = A_{k,2} \textbf{ And} \ldots \textbf{And } x_m = A_{k,m} \textbf{ Then } y = B_k \tag{10}$$

By introducing scaled distances on the consequence universe:

$$\delta_s(b_0, b_k) = \int_{b_0}^{b_k} s_Y(y)\,dy\ , \tag{11}$$

where s_Y is the i^{th} scaling function of the one dimensional consequent universe, b_k are the cores of the one dimensional fuzzy rule consequents B_k.

Introducing the first element of the one dimensional consequence universe b_0 (Y: $b_0 \leq y$, $\forall y \in Y$), based on (9) and (11), the requested one dimensional conclusion $y(\mathbf{x})$ can be obtained from the following formula [8], [11], [12]:

$$\delta_s(b_0, y(\mathbf{x})) = \begin{cases} \delta_s(b_0, b_k) & \text{if } \mathbf{x} = \mathbf{a}_k \text{ for some } k, \\ \left(\sum_{k=1}^{r} \delta_s(b_0, b_k) / \delta_{s,k}^{\lambda}\right) \Big/ \left(\sum_{k=1}^{r} 1/\delta_{s,k}^{\lambda}\right) & \text{otherwise.} \end{cases} \tag{12}$$

5 Fuzzy observation by merging vague environments

The lack of the fuzziness on the observation side in FIVE is inherited from the nature of the vague environment (see section 2), which describes the indistinguishability of two points and hence the Disconsistency Measure of a fuzzy set and a singleton only. For introducing fuzzy observation in FIVE, the concept of vague environment is needed to be extended to the observation too.

One possible solution for this task is a obvious one. If the observation is a fuzzy set, it can be also characterised by a vague environment in the same manner as it was done with the corresponding antecedent fuzzy partitions. This case the question of introducing fuzzy observation turns to be the question of merging two vague environments, the vague environment of the fuzzy observation and the corresponding antecedent fuzzy partition.

For merging two vague environments, the concept of *equal Disconsistency Measures* is applied. According to (3) the Disconsistency Measure of fuzzy sets A and B is the following:

$$S_D(A,B) = 1 - \sup_{x \in X} \mu_{A \cap B}(x) \;, \tag{13}$$

where $A \cap B$ notes the min t-norm, $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$ $\forall x \in X$.

Reconsidering the relation of the Disconsistency Measure of a fuzzy set and a singleton to the scaled distance of two values in a vague environment (according to (3)), the merged vague environment can be defined as the vague environment, where the scaled distance of two values is equal to the Disconsistency Measure of the two corresponding fuzzy sets (see e.g. on Fig.5):

$$S_D(A,B) = \left| \int_{\mathrm{a}}^{x_0} s_A(x)dx \right| = \left| \int_{x_0}^{\mathrm{b}} s_B(x)dx \right| = \left| \int_{\mathrm{a}}^{\mathrm{b}} s_{A'}(x)dx \right| = \delta_s(\mathrm{a},\mathrm{b}) = S_D(A',B') \;, \tag{14}$$

where $s_A(x)$ is the scaling function of fuzzy set A, $s_B(x)$ is the scaling function of fuzzy set B, $s_{A'}(x)$ is the merged scaling function on X and $\delta_s(\mathrm{a},\mathrm{b})$ is the scaled distance of the values a, b in $s_{A'}(x)$.

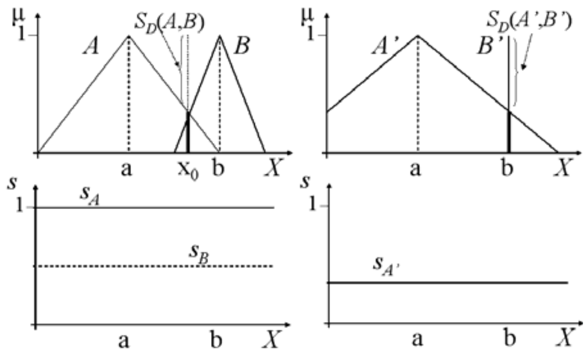

Fig.5. In the *merged scaling function* $s_{A'}$ (16), the scaled distance of two values a, b is equal to the Disconsistency Measure of the two corresponding fuzzy sets A, B.

Solving (14) in case of constant scaling functions (according to the notation of Fig.5), the following *merged scaling function* ($s_{A'}$) can be obtained:

$$S_D(A,B) = s_A \cdot (x_0 - \mathrm{a}) = s_B \cdot (\mathrm{b} - x_0) = S_D(A',B') \ , \tag{15}$$

$$S_D(A',B') = s_{A'} \cdot (\mathrm{b} - \mathrm{a}) = \frac{s_A \cdot s_B}{s_A + s_B} \cdot (\mathrm{b} - \mathrm{a}) \ \cdot$$

$$s_{A'}(x) = \frac{s_A(x) \cdot s_B(x)}{s_A(x) + s_B(x)} \qquad \forall \ \mathrm{x} \in \mathrm{X} \tag{16}$$

It is obvious, that generally (16) is not fulfilling the requirements of equal Disconsistency Measures for merging arbitrary vague environments, but it can serve as a kind of "approximation" for the merged scaling function.

Applying the concept of merged scaling function, the method FIVE can be simply completed by fuzzy observation. There is only one additional step required for the original method, the merging of the fuzzy observation vague environments to the vague environments of the corresponding antecedent fuzzy partitions. In the merged vague environment, the fuzzy observation turns to be a singleton, and hence the original FIVE method can be continued in the ordinary way.

Unfortunately, the vague environment merging of the fuzzy observations to the corresponding antecedent fuzzy partitions needed to be repeated in every reasoning step if the scaling function of the fuzzy observation is changing. On the other hand, in some cases, when all the observations can be characterised by the same scaling function (e.g. if all the fuzzy observations have the same isosceles triangle shaped membership function) the merging step needed to be completed only once for all the reasoning steps (see e.g. on Fig.6).

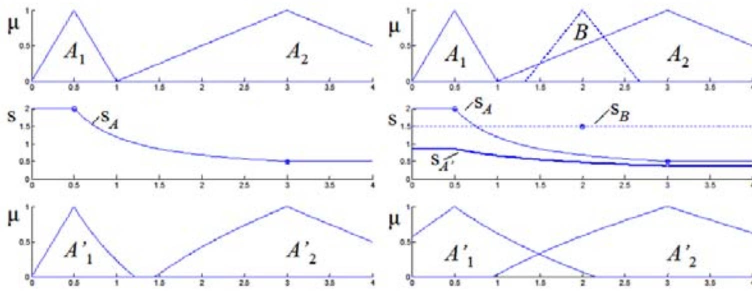

Fig.6. Fuzzy partition A_i described by scaling function s_A and the *merged scaling function* $s_{A'}$ constructed from s_A and s_B according to (16).

6 Example

Simple one-dimensional example for the fuzzy observation extended (16) FIVE method (12) is introduced in Fig.7. For comparing the crisp conclusions of FIVE to the classical methods, the conclusions generated by the max-min compositional rule of inference (CRI) and the centre of gravity defuzzification for the same rule base is also noted in the figure.

In Fig.7 the label "*fuzzy*" notes the case of fuzzy observation. For comparison, the figure also contains the conclusions of the crisp observations (label "*crisp*") for the same rule base. In the example it was assumed, that the running observation has the same isosceles triangle shaped membership function (see x on Fig.7) everywhere in the observation universe X. For the notation of the scaling function merging in Fig.7, see Fig.6.

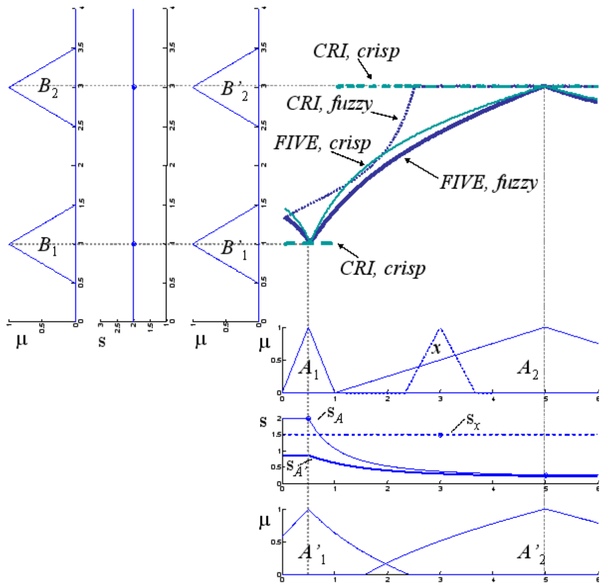

Fig.7. Interpolation of two fuzzy rules ($A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$) applying the fuzzy observation extended (16) FIVE fuzzy rule interpolation (12), λ=1.

7 Conclusions

The goal of this paper was to introduce a way for extending the "FIVE" FRI method to be able to handle fuzzy observations. The proposed extension, the "*vague environment merging*", unifies the vague environments of the fuzzy observations to the vague environments of the antecedent universes, and hence introduce the ability of handling fuzzy observation in FIVE.

The main drawback of the proposed extension is the additional step required for merging the observation and antecedent vague environments. This vague environment merging step is needed to be repeated in every reasoning case if the scaling function of the fuzzy observation is changing. On the other hand, when all the fuzzy observations can be characterised by the same scaling function, this merging step is needed to be done only once.

The merged vague environment is introduced as the vague environment, where the scaled distance of two values is equal to the Disconsistency Measure of the two corresponding fuzzy sets characterised by the two separate vague environments intended to be merged (see section 5 for more details). The function proposed for vague environment merging in this paper (16) is only a kind of approximation. Generally the requirement of equal Disconsistency Measure is not fulfilled, save the case when the scaling functions are constants. In spite of this drawback, the proposed merging function (16) is simple enough to keep the simplicity and reasoning speed of the fuzzy observation extended FIVE method. (Since the main goal of developing FIVE was to construct an FRI method, which could be implemented to be simple and quick enough to fit the requirements of real-time direct fuzzy logic control systems.)

A freely applicable code of the extended FIVE introduced in this paper, together with some application examples can be downloaded from [21].

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